1. Safety Aspects

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A structure must be designed to resist all loads expected to act on the structure during its service life. Under the effects of the expected applied loads, the structure must remain intact and perform satisfactorily. In addition, a structure must not require an inordinate amount of resources to construct. Thus, the design of a structure is a balance of necessary reliability and reasonable economy.

The safety requirements for the evaluation of a structure should:

- ***include all construction and design methods and all building materials, enabling comparison of different designs***
- ***make use of all available experience and expertise***
- ***be practical***

The first priority in the design process is to create a safe design, usually be following recognized rules or codes. The codes are continuously changing in order to utilize current techniques of design and analysis. The concept used in Canada at this time is the limit states approach. This approach uses safety factors for both the resistance and the load sides of the equation instead of only one global safety factor, as was done in former times in the working stress design method. The limit state design format bases the design of a structure on the probability and mode of failure, or limit usefulness, and the probability of the occurrence and variation of the load. Using this concept, the structure is characterized by its overall resistance, which leads to a better understanding of the structural behavior than working with arbitrary equivalent values which is the case with allowable stress design methods. For example, the failure of a column is described by the buckling resistance, rather than by an equivalent stress which relates to the buckling load.

Structural design must recognize that loads and resistances are really groups of data rather than single values. Like any group of data, there are statistical attributes such as mean, standard deviation, and coefficient of variation. Therefore, the design of a structure is a statistical problem since neither loads nor resistances are entirely predictable and can best be described by a statistical distribution. The probability of failure reflects a risk factor that the public is prepared to accept for a particular type of structure.

2. Theoretical Background

To determine the safety level of a structure, it is necessary to examine the populations of load and resistance values. The following statistical expressions are used to characterize groups of data.

- For a sample of n data points each with a value "x", the mean value is:

$$\bar{x} = \frac{1}{n} \Sigma x_i \qquad (1)$$

If $n \to \infty$ the mean value $\bar{x}$ becomes the mean m_x.

- The standard deviation is: $s_x = \sqrt{\frac{1}{n}\Sigma\left(x_i - \overline{x}\right)^2}$ (2)

  And if $n \to \infty$ the standard deviation becomes σ_x.

- The coefficient of variation, which characterizes the distribution of the mean, is for a population where $n \to \infty$ the following:

$$COV = V = \frac{\sigma_x}{m_x} \quad (3)$$

Samples of data can be characterized by mathematical models called distribution functions. Often a distribution is also expressed as the probability of a value being below a given value x. This probability is the integral of the distribution function. Distribution functions are called probability density function. Different data sets are better described by different types of distribution functions. While a normal distribution is suitable for dead loads, gamma distributions are often used for occupancy loads and snow loads are best modeled with a lognormal distribution.

Several functions are used in reliability analysis with the aim of obtaining the best possible fit. Many occurrences can be idealized best with a normal distribution curve which is also called Gaussian function.

3. The Normal Distribution (Gaussian)

The equation for the normal distribution by Gauss depends only on the values m_x and σ_x:

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2\right] \quad (4)$$

In Figure 1 the normal distribution function *f(x)* of the characteristic value x can be seen. The mean value m_x represents the centroid of the area below the function *f(x)*.
The standard deviation can be recognized as the radius of gyration as measured from m_x. The area beneath the function *f(x)* is defined as unity.

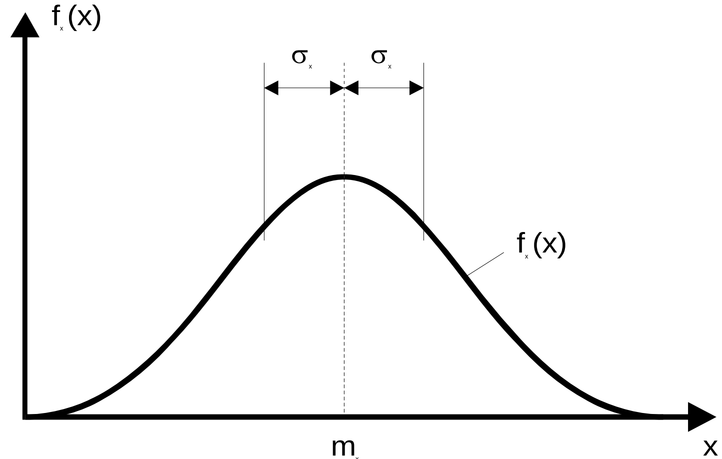

Figure 1: Normal Distribution Curve

The cumulative distribution function, *F(x)*, which can be seen in Figure 2, is the probability that the variable takes a value less than or equal to x. The correlation between the cumulative distribution function and the density function is the following:

$$F(x) = \int_{-\infty}^{x} f_x(x)\,dx$$

Or

$$\frac{dF(x)}{dx} = f(x) \qquad (5)$$

For example the function value $F(x_1)$ indicates what fraction of the population is smaller or equal to the value x_1. It can be seen as the probability p that the value x is not larger than x_1.
Thus, together with equation (5), the probability of an event occuring can be derived (i.e., the value of x to be smaller or equal to x_1):

$$p = F(x_1) = \int_{-\infty}^{x_1} f(x)\,dx \qquad (6)$$

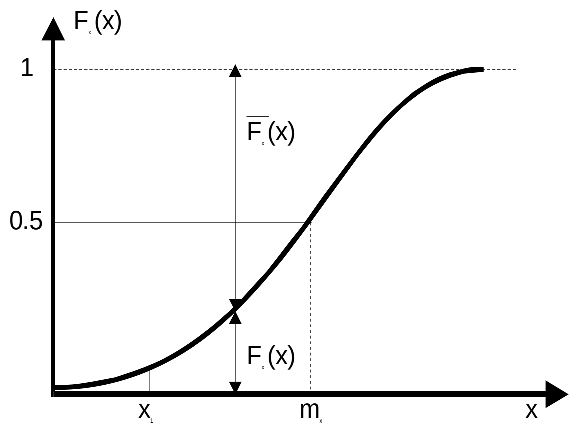

Figure 2: Cumulative Distribution Curve

The Gaussian function cannot be integrated in closed form. The function values *f(x)* and the integrals *F(x)* are usually provided in tabular form. Often the concept of percentile x_p is used, e.g., the 5th percentile or the 95th percentile. It has the meaning that p% of the events are expected to be below this value.

In a normal distribution the relationship between the percentile value x_p (expressed as p [%]) and the standard deviation σ_x is as follows:

$$x_p = m_x \pm k * \sigma_x \qquad (7)$$

And can be tabulated (see Table 1).

x_p [%]	k
20	0.842
10	1.282
5	1.645
2.50000	1.960
2.27500	2.000
1.00000	2.326
0.13500	3.000
0.00320	4.000
0.00003	5.000

Table 1: Percentile and their k-Values

For example, the 5th-percentile is the point that 5 [%] of values are expected to be below. This can be seen in Figure 3:

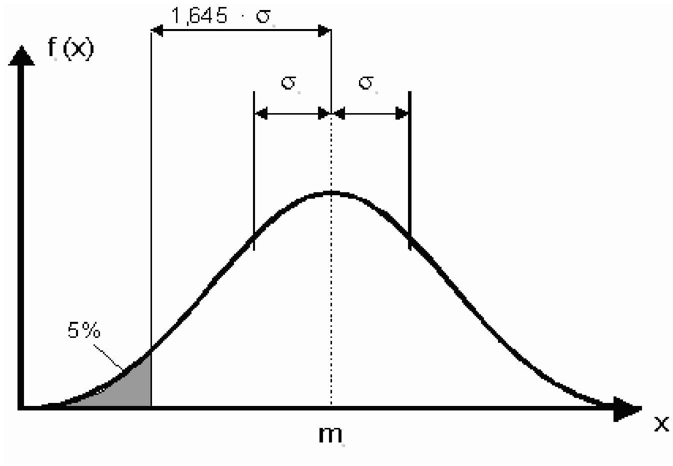

Figure 3: 5th-Percentile

4. The Probability of Failure – Safety Margins

The probability of failure is related to the overlapping area (marked grey in Figure 4) of the load and resistance distributions. Since it is very difficult to obtain information about the tail ends of the populations, statistical methods have to be used to establish an acceptable safety standard.

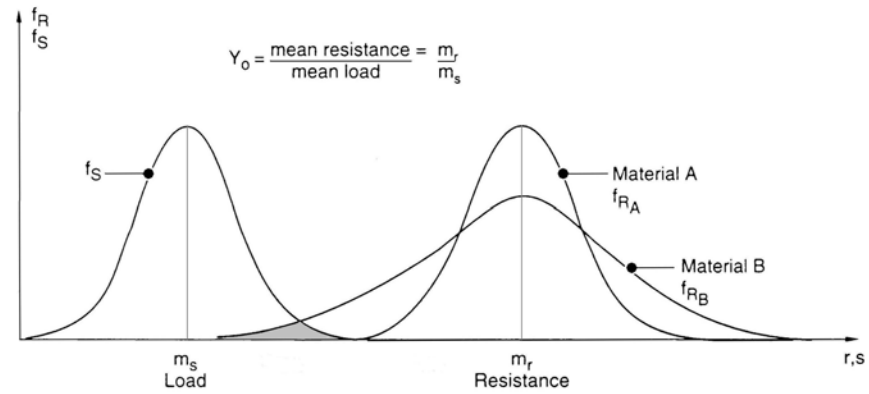

Figure 4: The Global Safety Factor

The simplest failure measure is the global safety factor γ_o, which was used in an older design method, working stress design. The global safety factor is equal to the ratio between the mean value of resistance and the mean value of load.

$$\gamma_o = \frac{m_R}{m_S} \qquad (8)$$

This distribution gives a poor indication of the possibility of failure since it does not include any information on the spread of the distribution, and a wide variety of safety margins are possible.
In fact, for distributions having the same global safety factor, γ_o, the occurrence of failure will be inconsistent because the actual safety depends not only on the mean values, but also on the variability in the global safety factor.

Thus, in order to achieve a consistent probability of failure, the global safety factor will have to be adjusted depending on the scatter of load and resistance values.

The nominal safety factor, γ_p, is a more accurate way to describe the overlapping area of the load and resistance curves. It is the ratio of a certain percentile of the resistance, r_p, to a certain percentile of the load, s_p:

$$\gamma_p = \frac{r_p}{s_p} \qquad (9)$$

For normal distributions, the 95th percentile of the load and the 5th percentile of the resistance are typically used values for structural design and result in:

$$\gamma_p = \frac{r_{5\%}}{s_{95\%}} \qquad (10)$$

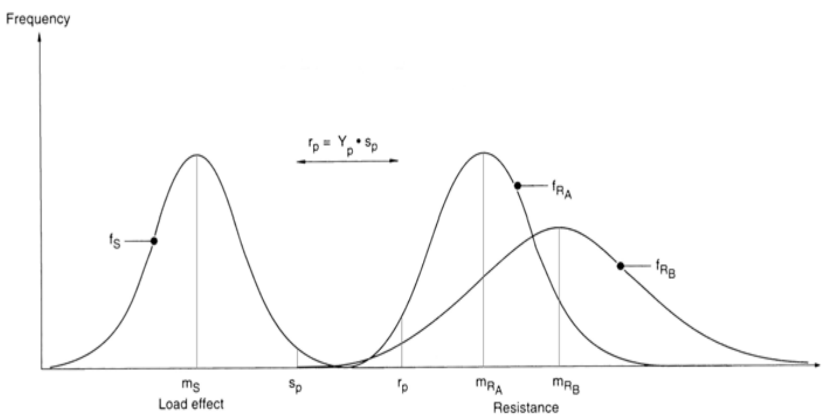

Figure 5: Nominal Safety Factor

5. Safety Index

A consistent probability of failure can be assumed by using a probability density distribution, f_z, of the safety parameter Z, which is obtained by subtracting the load S (like stress) from the resistance R for every possible combination of R and S (see Figure 5). The structure is deemed to be safe if the safety margin Z is:

$$Z = R - S \qquad (11)$$

Therefore the defined limit state is reached when

$$R = S \quad \text{or rather} \quad Z = 0 \qquad (12)$$

This results in a new distribution of the performance parameter Z. All negative values of the function $f_z(z)$ indicate failure, while positive values indicate success. The probability of failure is thus represented by the area underneath the curve and left of the origin, given by:

$$P_f = \int_{-\infty}^{0} f_z(z)\,dz \qquad (13)$$

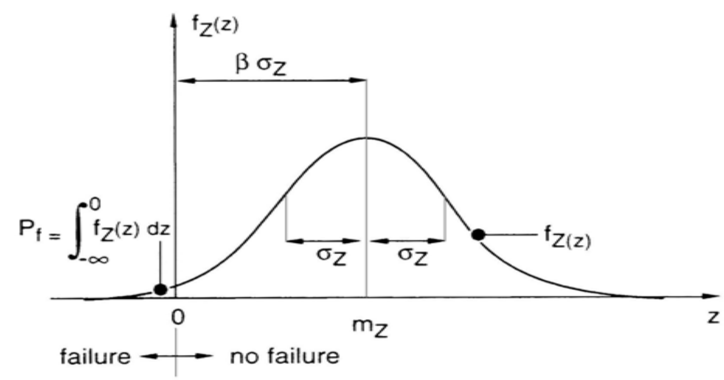

Figure 6: Normal Distribution Density of the Safety Zone Z

The load S and the resistance R are independent quantities with randomly scattered magnitudes described by normal distribution densities $f_S(s)$ and $f_R(r)$, mean values m_S and m_R and standard deviations σ_S and σ_R. Thus, since $f_S(s)$ and $f_R(r)$ are normally distributed, then $f_Z(z)$ is also a normal distribution. Accordingly the mean value m_z and the standard deviation s_z are obtained from the load and resistance distributions as follows:

$$m_z = m_R - m_s \tag{14}$$

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{15}$$

These values can be seen in Figure 7.

The limit state equation (11) is valid for all the random values s and r corresponding to the action and resistance for which:

$$r = s \quad \text{or} \quad z = 0 \tag{16}$$

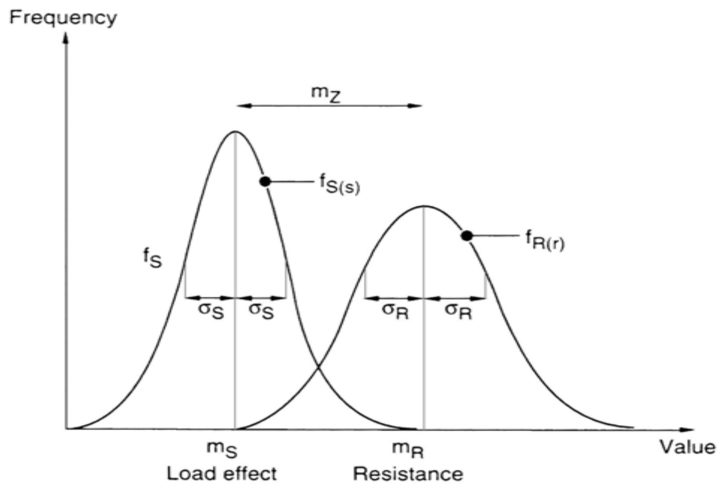

Figure 7: Normal Distribution Density of Load and Resistance

The inverse coefficient of variation of the function $f_Z(z)$ is defined as the safety index:

$$\beta = \frac{m_z}{\sigma_z} = \frac{m_R - m_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \tag{17}$$

The safety index β characterizes the probability of failure in a more appropriate way than the safety factor γ, because it takes the distribution of load and resistance into account. In other words, equal values of β mean equal degrees of reliability. The case of failure can be interpreted as a percentile value of the distribution density function $f_Z(z)$. An increasing β thus correlates with a higher safety. The appropriate safety index is chosen in accordance with the consequence of failure and should be the same for all materials. In North American Codes, a value of β between 2.4 and 2.9 is generally considered acceptable.

By calculating β-values for existing structures that have been designed using common specifications, it can be assessed what β-values are tolerated by the general public. The following numerical values seem to correspond with normal practice:

	Safety Index β		
Limit State	Degree of Damage		
	small	medium	severe
Serviceability	2.0	2.5	3.0
Ultimate	4.2	4.7	5.2

Table 2: b-values tolerated by the general public

For linear problems which obey first order theory and have a normal distribution, the following relationship exists between the safety index β and the probability of failure p_f:

β	p_f
5.2	10^{-7}
4.7	10^{-6}
4.2	10^{-5}
3.7	10^{-4}
3.2	10^{-3}

Table 3: Safety index b and the corresponding probability of failure for linear problems

The failure probability represents the sum of all occurrences which have a negative value of the function $f_z(z)$. For example, a failure probability of 0.001 = 0.1 [%] ($p_f = 10^{-3}$) means that for every 1000 structures designed and built with a safety index of 3.2, one failure can be expected.

6. Relationship between Safety Index β and Safety Factor γ

The purpose of the design process is to maintain an acceptable difference between the action S and the resistance R.

For obvious reasons, a high action percentile value is used for the load side and a low value for the resistance side; for example the common values used are 95 [%] for the load and 5 [%] for the resistance.

$$R_P = m_R - k_R \sigma_F \qquad (18)$$

$$S_q = m_s + k_s \sigma_s \qquad (19)$$

Equations 18 and 19 are only valid for normal distributions.

The general safety in design, γ is defined by the following (with Equations 18 and 19):

$$\gamma = \frac{R_p}{S_q} = \frac{m_R - k_R \sigma_R}{m_S + k_S \sigma_S} \qquad (20)$$

Equation 17 can be rewritten as the following:

$$\beta(\sqrt{\sigma_R^2 + \sigma_s^2}) = m_R - m_s$$

$$\beta\left(\frac{\sigma_R^2 + \sigma_s^2}{\sqrt{\sigma_R^2 + \sigma_s^2}}\right) = m_R - m_s \qquad (21)$$

This equation can be simplified significantly by applying the following "linearization" procedure:

$$\sqrt{\sigma_R^2 + \sigma_S^2} = \alpha_R \sigma_R + \alpha_S \sigma_S \qquad (22)$$

Where:

$$\alpha_R = \frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (23)$$

$$\alpha_S = \frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (24)$$

This "linearization" is the only approximation which is used in the safety concept. It follows from Equation 21 that:

$$\beta(\alpha_R \sigma_R + \alpha_S \sigma_S) = m_R - m_S \qquad (25)$$

If all of the resistance is on one side and the entire load on the other, with the help of Equation 3, Equation 25 can be rewritten as the following:

$$m_R - \beta \alpha_R \sigma_R = m_S + \beta \alpha_S \sigma_S$$

$$m_R (1 - \beta \alpha_R V_R) = m_S (1 + \beta \alpha_S V_S) \qquad (26)$$

Since the global safety factor is defined by the ratio of the means we can rewrite equation 8 with the results of Equation 26:

$$\gamma_o = \frac{m_R}{m_S} = \frac{1 + \beta \alpha_S V_S}{1 - \beta \alpha_R V_R} \qquad (27)$$

If we now take Equation 20 and rewrite it so that the ratio of the means occurs separately, the result will be the following:

$$\gamma = \frac{m_R - k_R\sigma_R}{m_S + k_S\sigma_S} = \frac{m_R}{m_S}\frac{(1-k_R V_R)}{(1+k_S V_S)} \qquad (28)$$

And now Equation 27 can be used to express the ratio of the means in Equation 28:

$$\gamma = \frac{1+\beta\alpha_S V_S}{1-\beta\alpha_R V_R} * \frac{(1-k_R V_R)}{(1+k_S V_S)} \qquad (28)$$

Re-arranging Equation 28 results in the following:

$$\gamma = \frac{1+\beta\alpha_S V_S}{(1+k_S V_S)} * \frac{(1-k_R V_R)}{1-\beta\alpha_R V_R} \qquad (29)$$

$$\gamma = \quad \gamma_S \quad * \quad \gamma_R$$

Hence the partial safety factors are:

$$\gamma_S = \frac{1+\beta\alpha_S V_S}{1-k_S V_S} \qquad \text{For the loads} \qquad (30)$$

$$\gamma_R = \frac{1-k_R V_R}{1+\beta\alpha_R V_R} \qquad \text{For the resistance} \qquad (31)$$

By introducing these factors, the design process can be expressed by the following:

$$\gamma_S S_q \leq \frac{R_p}{\gamma_R} \qquad (32)$$

In other words, the overall nominal safety factor γ[1] is split up into a partial safety factor γ_S and a partial safety factor γ_R. γ_S is multiplied by the characteristic value of the action (or percentile), which is independent of the material (resistance). Similarly, γ_R, which is divided by the characteristic value of the resistance (or percentile), is independent of the action (load).

An advantage of a design code based on safety index and limit states design, like the Canadian Code, is that the probability of failure for different loading conditions is made more consistent. This is achieved by the use of distinct load factors for the different loads a structure is subjected to. In working stress design, only a single factor of safety was used. Furthermore, different resistance factors can be applied in parallel manner to determine member resistances with uniform reliability. The combination of the load factor and the inverse of the resistance factor gives a number comparable to a global safety factor.

7. Loads and Safety Criterion

The fundamental safety criterion that must be met, namely:

Factored Resistance ≥ Effect of Factored Loads,

or

$$\boldsymbol{\phi R \;\geq\; \alpha_D D + \gamma\,\psi\,(\alpha_L L + \alpha_W W + \alpha_T T)}$$

For all load combinations including those involving earthquake, the expression for the *effect of factored loads* is identical with that given in Part 4 of the National Building Code of Canada (NBCC) as are the values given for the **various load factors (α)** a **load combination factors (ψ)** and **importance factors (γ)**.

The factored resistance is given by the product ϕR where ϕ the resistance factor is and R is the nominal member strength, or resistance.

[1] Note:
In reality there is a dependence of the two partial safety factors, since the two values α_R and α_s are linked together ($\alpha^2_R + \alpha^2_S = 1$).