CASE STUDIES OF VARIOUS FRAMED STRUCTURES

– Elastic and Plastic Analysis Using Dr. Frame

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ABSTRACT

This thesis is to present the elastic and plastic analysis of various framed structures using Dr. Frame. The basic concepts of elastic and plastic analysis are introduced, as well as the fundamental feature and algorithm of Dr. Frame. The analysis results of five different frames are demonstrated, such as the collapse load factor λ_c, the sketch of the collapse mechanism, etc.

INTRODUCTION

It is more complicated to do the plastic analysis of a frame with high degree of redundancy since it is not practical to discuss all possible collapse mechanisms. But with the aid of advanced computer program, such as Dr. Frame, the sufficient accurate result can be achieved. In this thesis, five different frame cases are considered ranging from one-bay single-story frame to two-bay multistory frame.

First, the basic concepts of elastic and plastic analysis are briefly illustrated, and the main focus is on the plastic hinge, plastic moment and the fundamental theorems of finding the collapse load factor λ_c.

Second, the essential feature and algorithm of plastic analysis in Dr. Frame are introduced, such as plastic hinge objects, static linear and second order analysis involving $P-\Delta$ effects and load-dependent EI.

Third, the results of the elastic and plastic analysis of five study frames are demonstrated. For each case, the collapse load factor λ_c and the collapse mechanism are obtained by performing Dr. Frame. Also, according to the analysis results some comparison and discussion are done. For example, the plastic moment reduction due to axial forces, as well as the linear analysis result compared to the second order analysis result for single-story frame and multistory frame.

BASIC CONCEPT

1.1 Elastic Analysis

The behavior of steel when loaded below the yield point is much closer to the true elastic behavior than that of other structural materials. In the elastic range all elements and the complete structure are assumed to obey Hooke's law and recover to their original state on removal of load.

For framed structures, linear (first-order) elastic theory is traditionally used for analysis. With the aid of computer program, second-order analysis taking account of deflections in the structure can be performed. The maximum elastic load capacity is determined when any point in any member section reaches the yield stress or elastic critical buckling stress where stability is a problem.

1.2 Plastic Analysis

When a steel specimen is loaded beyond the elastic limit the stress remains constant while the strain increases. For a beam or column subjected to increasing moment this behavior results in the formation of a plastic hinge where a section rotates at the plastic moment capacity.

Plastic analysis is based on determining the minimum load that causes the structure to collapse. Collapse occurs when sufficient plastic hinges have formed to convert the structure to a mechanism.

The plastic analysis for frames assumes that all rotational deformation occurs in the plastic hinges. During loading, framed structure first acts elastically and, as the load increases, hinges form successively until the frame is converted to a mechanism. More accurate second-order analyses take the frame deflections into account.

1.3 Plastic Hinge

Fully understanding the concept of plastic hinge is very important. Because the word hinge suggests free rotation, while here rotation can only occur when the bending moment reaches the plastic moment capacity. In the neighborhood of a section of maximum bending moment, a region of localized plastic deformations forms in which the curvature is much larger than elsewhere. For analyzing the behavior of frames, it is convenient and sufficiently accurate to assume that plastic hinges appear in sections of maximum bending moments. These plastic hinges, however, do not permit a free relative rotation of the parts they join, but resist this rotation with a constant plastic moment capacity.

1.4 Plastic Moment

The plastic moment plays an important role in the plastic analysis. It is, theoretically, the biggest bending moment that a section can withstand. The simple formula for the plastic moment is $M_p = ZF_y$, but in many section it turns out to be impossible to reach this value due to axial force, shear force, etc.

1.4.1 Effect of the Axial Force

Columns may have to carry significant axial forces in addition to bending moment. The axial forces N, both tensile and compressive, reduce the plastic moment.

The formulas used to calculate the reduced plastic moment are listed below.

- Rectangular Cross Section:

$$\frac{M}{M_p} = 1 - \left(\frac{N}{N_p}\right)^2$$

- I Cross Section subjected to bending with respect to its “Strong Axis”:

$$\frac{M}{M_p} = 1 - \left(\frac{N}{N_p}\right)^2 \frac{A^2}{4wZ_x} \qquad \text{for} \qquad \frac{N}{N_p} < \frac{A_w}{A}$$

$$\frac{M}{M_p} = \frac{A}{2Z_x}\left(1 - \frac{N}{N_p}\right)\left\{h - \left[A\left(1 - \frac{N}{N_p}\right)2b\right]\right\}$$

$$\text{for} \qquad \frac{A_w}{A} \le \frac{N}{N_P} \le 1$$

- I Cross Section subjected to bending with respect to its "Weak Axis":

$$\frac{M}{M_p} = 1 - \left(\frac{N}{N_p}\right)^2 \frac{A^2}{4hZ_y} \qquad \text{for} \qquad 0 \le \frac{N}{N_p} \le \frac{wh}{A}$$

$$\frac{M}{M_p} = \frac{A^2}{8tZ_y}\left[\frac{4bt}{A} - \left(1 - \frac{N}{N_p}\right)\right]\left(1 - \frac{N}{N_p}\right)$$

$$\text{for} \qquad \frac{wh}{A} \le \frac{N}{N_P} \le 1$$

M : reduced plastic moment

M_p : simple plastic moment, $= ZF_y$

N : axial force

N_p : maximum axial force, $= AF_y$

A : cross section area

A_w : web area

Z_x : plastic section modulus with respect to "strong axis"

Z_y : plastic section modulus with respect to "weak axis"

b, h, t, w : cross section parameters, shown in Figure 1.1

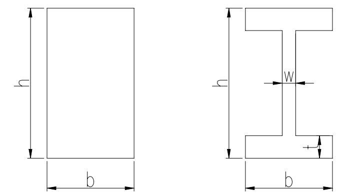


Figure 1.1 Cross Section Parameters

1.4.2 Effect of the Shear Force

Generally, beams must resist a combination of a bending and a shear force. This means there is a combination of direct stress σ from the moment and shear stress τ. In these circumstances, the shear and direct stresses interact with each other in causing the material to yield. In order to determine the on set of yield, a yield criterion has to be used. The Tresca and Von-Mises yield criteria are most commonly used for ductile materials, such as steel.

The research indicated that shear forces cause smaller reductions in the plastic moment than axial forces, and need only be considered when they are exceptionally large. In I beam, the reduction in the plastic moment is negligible until the shear force is over 50% shear capacity of the web. Reductions from shear forces are only likely to be significant in short span beams carrying very large concentrated loads.

1.5 Fundamental Theorems of Finding Collapse Load Factor λc

The general methods of finding collapse load factor are based on three fundamental theorems, called the static theorem, the kinematic theorem and uniqueness theorem. The first theorem furnishes a lower boundary for λ_c, and the second one provides an upper boundary. Combined with the first two theorems, the third theorem gives the actual value of λ_c.

1.5.1 Static Theorem

If there exists a bending moment distribution throughout a frame structure which is both safe and statically admissible for a set of loads λP, then the value of λ must be less than or equal to the collapse load factor λ_c.

$$\lambda \le \lambda_c$$

1.5.2 Kinematic Theorem

For a given frame structure subjected to a set of loads λP, the value of λ which corresponds to the assumed mechanism must be greater or equal to the collapse load factor λ_c.

$$\lambda \ge \lambda_c$$

1.5.3 Uniqueness Theorem

For a given frame structure and a set of loads λP, if there is at least one safe and statically admissible bending moment distribution, in which the plastic moment occurs at enough cross sections to produce a mechanism, the corresponding load factor will be the collapse load factor λ_c.

$$\lambda = \lambda_c$$

However, for a framed structure of complicated shape and with a high degree of redundancy, it is not possible to assume and discuss all possible collapse mechanisms, and the above three theorems are no longer applicable. In such situation, an advanced computer program will be very helpful to find the collapse load factor λ_c.

1.6 Clauses Excerpt from CSA-S16.1

The clauses in the CAN/CSA-S16.1 relating to the elastic and plastic analysis are as followed.

Clause 8.4 Elastic Analysis

Under a particular loading combination, the forces and moments throughout all or part of the structure may be determined by an analysis that assumes that individual members behave elastically.

Clause 8.5 Plastic Analysis

Under a particular loading combination, the forces and moments throughout all or part of the structure may be determined by a plastic analysis provided that

(a) the steel used has $F_y \le 0.80 F_u$ and exhibits the load-strain characteristics necessary to achieve moment redistribution;

(b) the width-thickness ratios meet the requirements of Class 1 sections as given in Clause 11.2;

(c) the members are braced laterally in accordance with the requirements of Clause 13.7;

(d) web stiffeners are supplied on a member at a point of load application where a plastic hinge would form;

(e) splices in beams or columns are designed to transmit 1.1 times the maximum calculated moment under factored loads at the splice location or $0.25M_p$, whichever is greater;

(f) members are not subject to repeated heavy impact or fatigue; and

(g) the influence of inelastic deformation on the strength of the structure is taken into account.

ABOUT DR. FRAME

As mentioned before, it is no longer possible to assume and discuss all possible collapse mechanisms for a framed structure with high degree of redundancy. This leads to using computer program to do the plastic analysis. In this thesis, Dr. Frame is the main tool to do the elastic and plastic analysis of various framed structures. The fundamental feature and algorithm of this program are briefly introduced as following.

2.1 Plastic Hinge Objects

Dr. Frame provides the unique ability to model and observe plastic behavior in structures interactively. The installation and removal of plastic hinges is the key to modeling plastic behavior in frames using Dr. Frame. Automatic plastic hinges will become activated when the moment acting at the hinge's location exceeds $M_p = ZF_y$, in which Z and F_y are taken from the property settings for each member in effect at the time the hinges were installed. Plastic hinges display themselves as blue rectangles prior to yielding. Following yielding, they display themselves as either solid red circles during loading or open red circles during unloading.

Dr. Frame exploits the fact that any nonlinear analysis can be treated as approximately linear solution. In fact, for the case of a structure with discrete, bilinear yield mechanisms and otherwise linear response, the behavior can be treated as exactly piecewise linear. To model hinge formation and the subsequent yielded response of a plastic hinge, Dr. Frame uses plastic hinge objects. Each plastic hinge does the following:

- Knows which member it belongs to, and where it is located on the member.
- Knows what moment is necessary to cause itself to yield (or what combination of moment and axial force will cause yield).
- Installs itself as a regular hinge combined with applied moments when active and loading.
- Keeps track of the previously applied moment at its location. This enables the plastic hinge to determine moment increments from one load step to the next.
- Watches each increment of rotation to detect unloading.
- When unloading, installs itself as a locked up (rigidly constrained) hinge with an imposed rotation jump. The corresponding constraint equation is of the form $\theta_{left} = \theta_{right} + \theta_p$.

- The plastic hinge yield criterion uses a simple linear interaction relation to account for axial force effect. The relation is of the form $\frac{N}{N_p} + \beta \frac{M}{M_p} = 1$, in which β is chosen according to the level of axial force, as outlined in the AISC LRFD specification.

These capabilities allow the plastic hinge objects to independently manage themselves, and to take responsibility for most the detail of the inelastic analysis.

2.2 Second-Order Analysis

In addition to basic linear analysis, Dr. Frame can perform second-order analysis involving nonlinear geometric effects. Its modeling of geometric nonlinearities includes the effects of axial force and bending moment interactions within individual members, and the overall influence of loading applied to a deformed structure.

Dr. Frame performs 2nd-order analysis in the following manner:

- A standard linear solution is computed, and the resulting axial forces, N, in each member are determined.
- The system is analyzed again using modified member stiffness based on the solution to the following differential equation and boundary conditions:

$$EIv^{'} + Nv = w_0 + w_0^{'} x$$

$$v(0) = v_0 ; v^{'}(0) = \theta_0$$

$$v(l) = v_l ; v^{'}(l) = \theta_l$$

The solution to this equation is easily obtained (see e.g., Timoshenko & Gere's Theory of Elastic Stability).

2.3 Load-Dependent EI

To approximately model more realistic axial capacities and member behavior than those predicted by a strictly elastic analysis, Dr.Frame chooses to have the effective EI of each member reduced as a function of its axial load. Each member's effective EI is computed as follows:

$$EI_{eff} = \phi_c EI \times \begin{cases} \frac{N}{N_y} \ln\left(\frac{N}{N_y}\right) \frac{1}{\ln(0.658)} & ; \frac{N}{N_y} > 0.38995 \\ 0.877 & ; \frac{N}{N_y} \le 0.38995 \end{cases}$$

in which N_y represents the simple yield capacity of the section, N is the axial load present in the member,

Just as in a real structure, load-dependent bending stiffness significantly alters the member's ability to carry load and to contribute stiffness to the rest of the structure. For practical analyses involving steel members, Dr. Frame recommends that using load-dependent EI whenever second-order analysis is performed.

CASE STUDIES

3.1 Case Model

Five different frame cases are considered in this thesis. In order to do some analysis result comparison, there are many load and geometric similarities from case to case. For example, the beam span is 8m and the story height is 4m in all five cases. Also all beam members are W310 x 60 and columns are W250 x 67. All structure members are Class 1 section, and supposed to be braced laterally, which prevent the overall structure buckling. Figure 3.1 illustrates the model of five study cases.

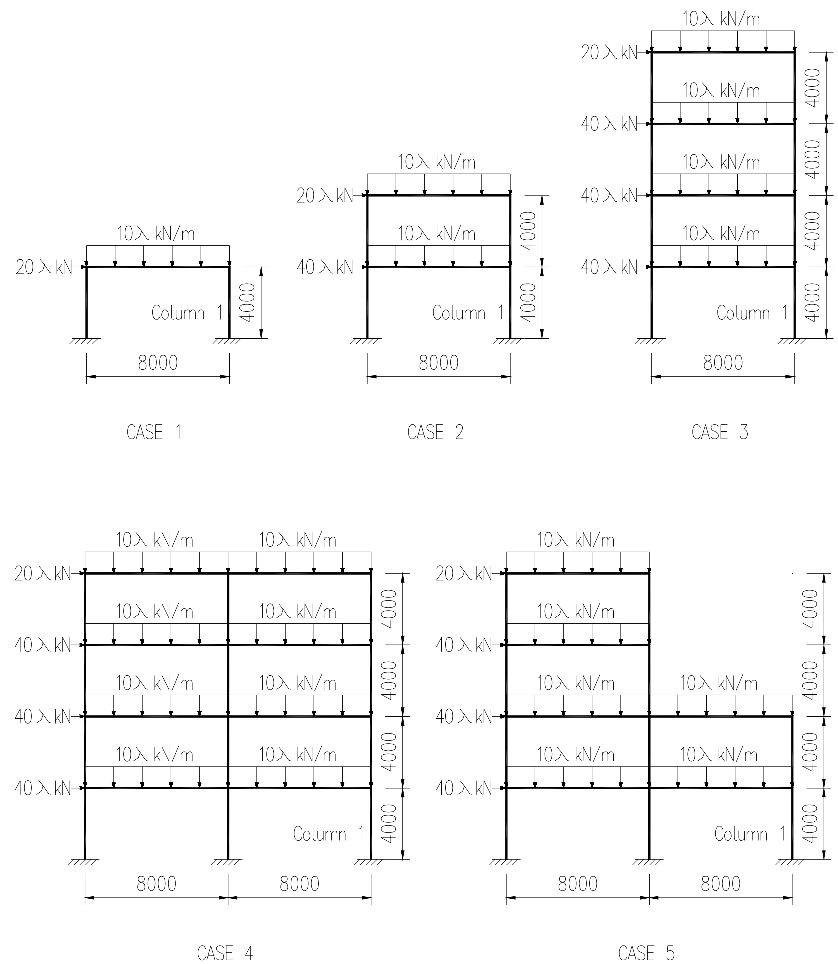


Figure 3.1 Model of Cases

3.2 Analysis Result

By performing Dr. Frame, the elastic limit load factor λ_e and the plastic collapse load factor λ_c can be achieved for each case, as well as the collapse mechanism.

Case 1: It is a one-bay single-story frame with $\lambda_e = 5.35, \lambda_c = 7.70$. It collapses due to a combined mechanism when the 4th plastic hinge forms.

Case 2: It is a one-bay two-story frame with $\lambda_e = 3.11, \lambda_c = 4.73$. It collapses due to a combined mechanism when the 6th plastic hinges form. Actually, there are two 6th plastic hinges in this case since they form simultaneously.

Case 3: It is a one-bay four-story frame with $\lambda_e = 1.43, \lambda_c = 1.95$. It collapses due to a sway mechanism when the 7th plastic hinge forms.

Case 4: It is a two-bay four-story frame with $\lambda_e = 2.07, \lambda_c = 2.96$. It collapses due to a sway mechanism when two 14th plastic hinges form.

Case 5: It is an asymmetric two-bay frame, one bay with four stories and the other with two stories. Its load factor is $\lambda_e = 2.12, \lambda_c = 3.06$. It collapses due to a sway mechanism when two 14th plastic hinges form.

The analysis results are illustrated in figure 3.2 ~ figure 3.6.

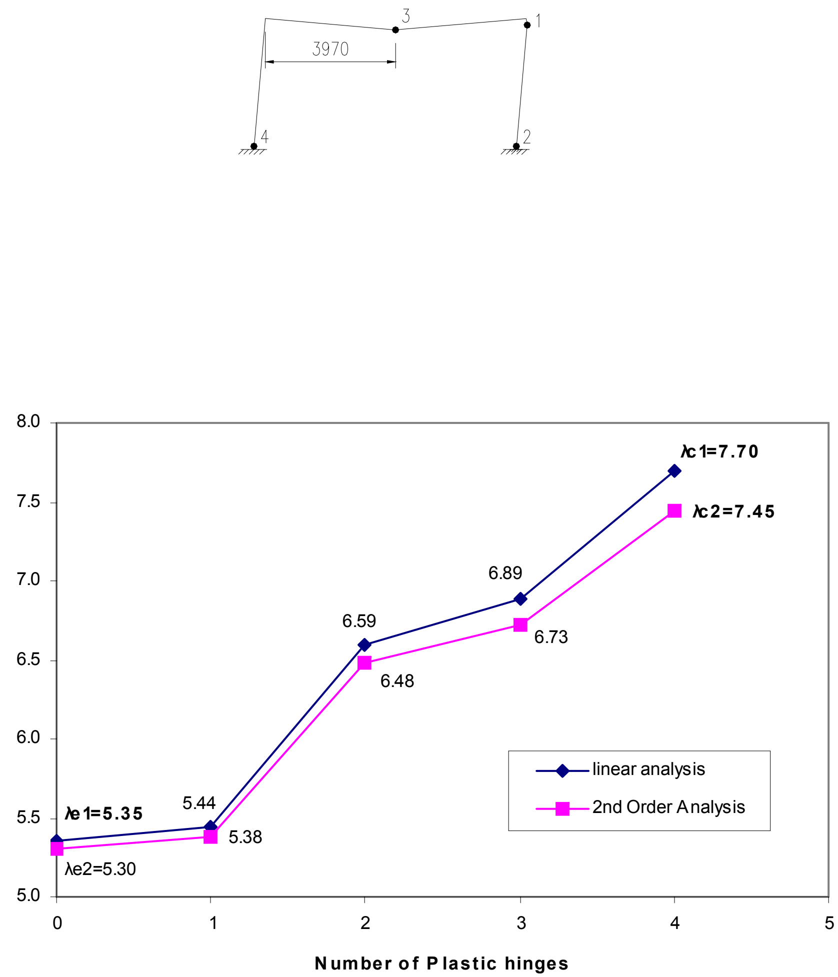


Figure 3.2 Collapse Mechanism and Load Factor λ in Case 1

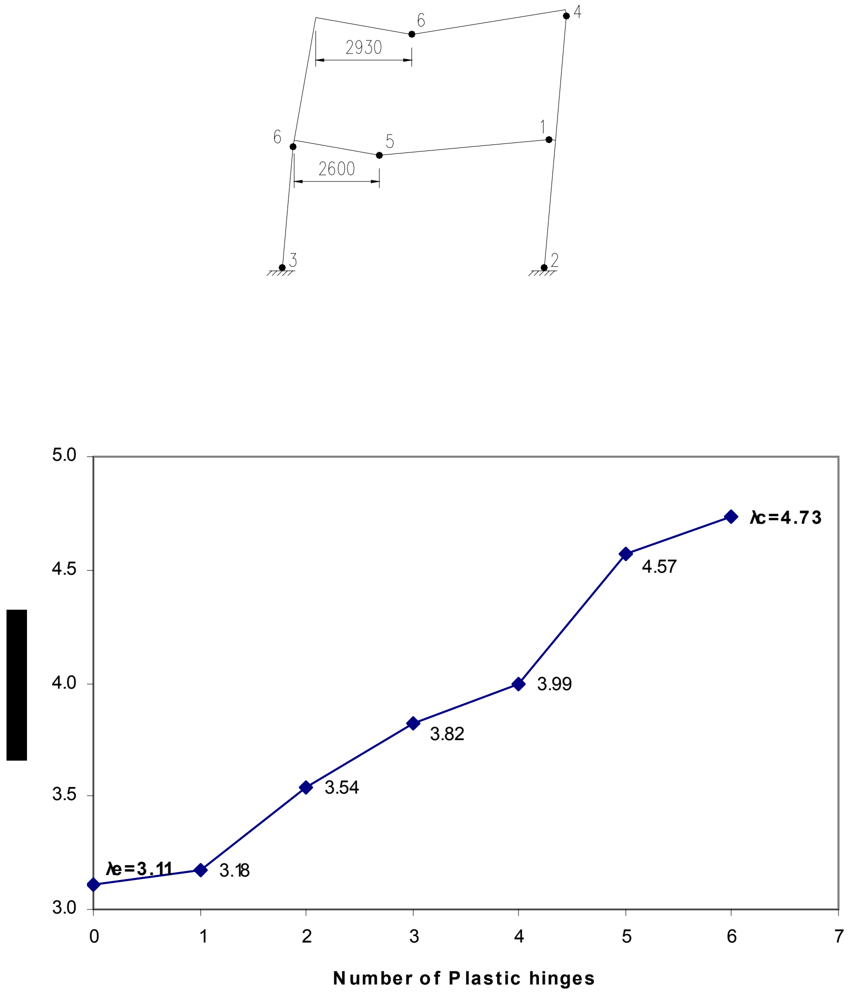


Figure 3.3 Collapse Mechanism and Load Factor λ in Case 2

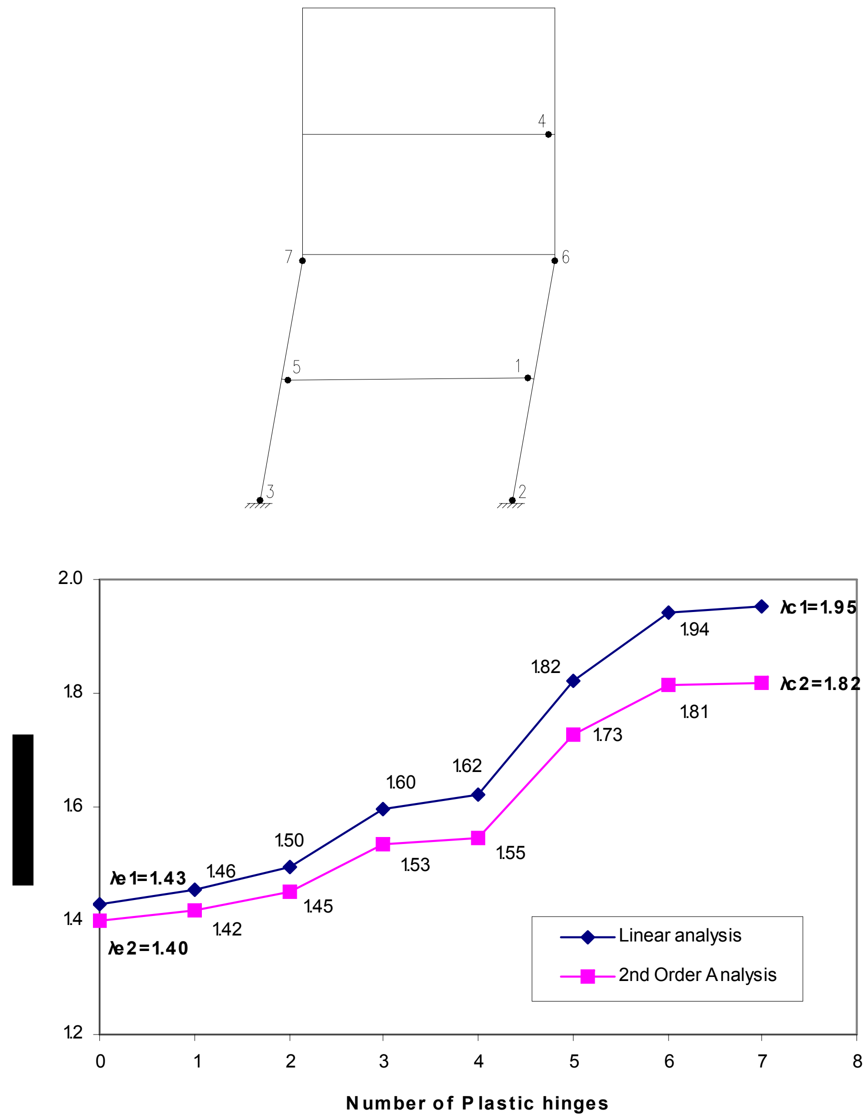


Figure 3.4 Collapse Mechanism and Load Factor λ in Case 3

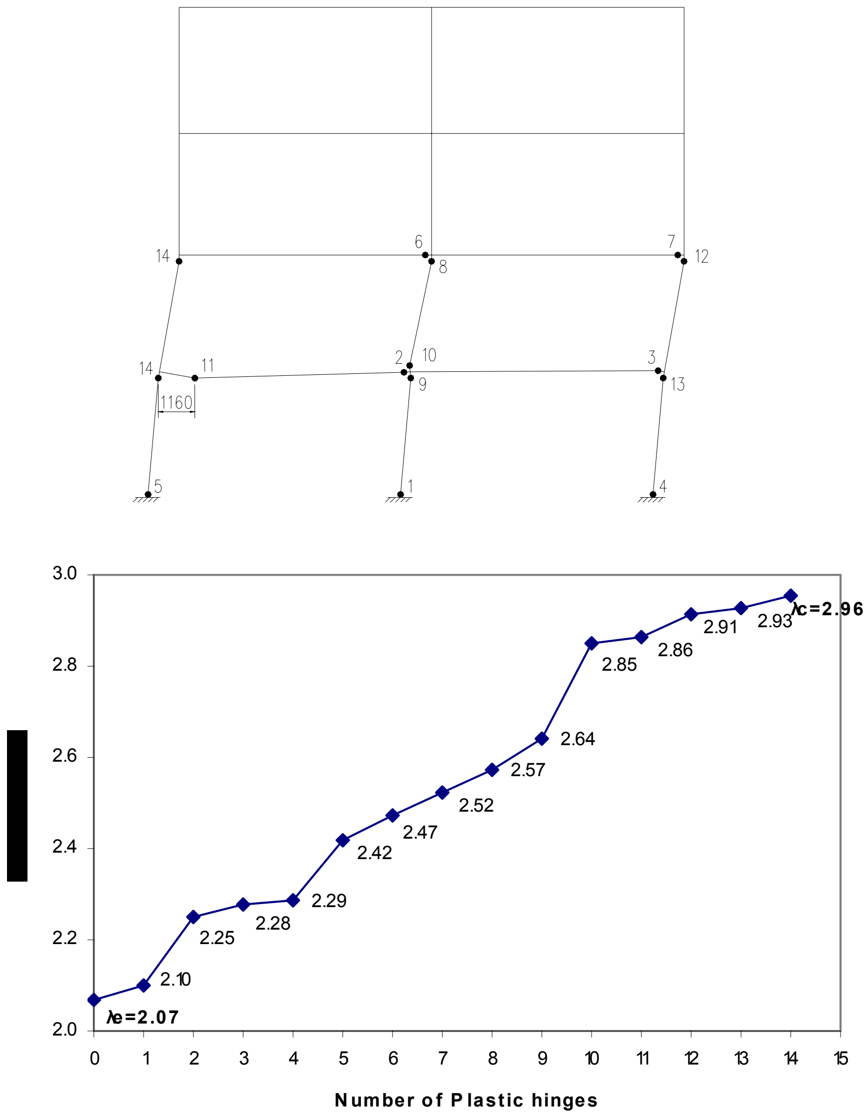


Figure 3.5 Collapse Mechanism and Load Factor λ in Case 4

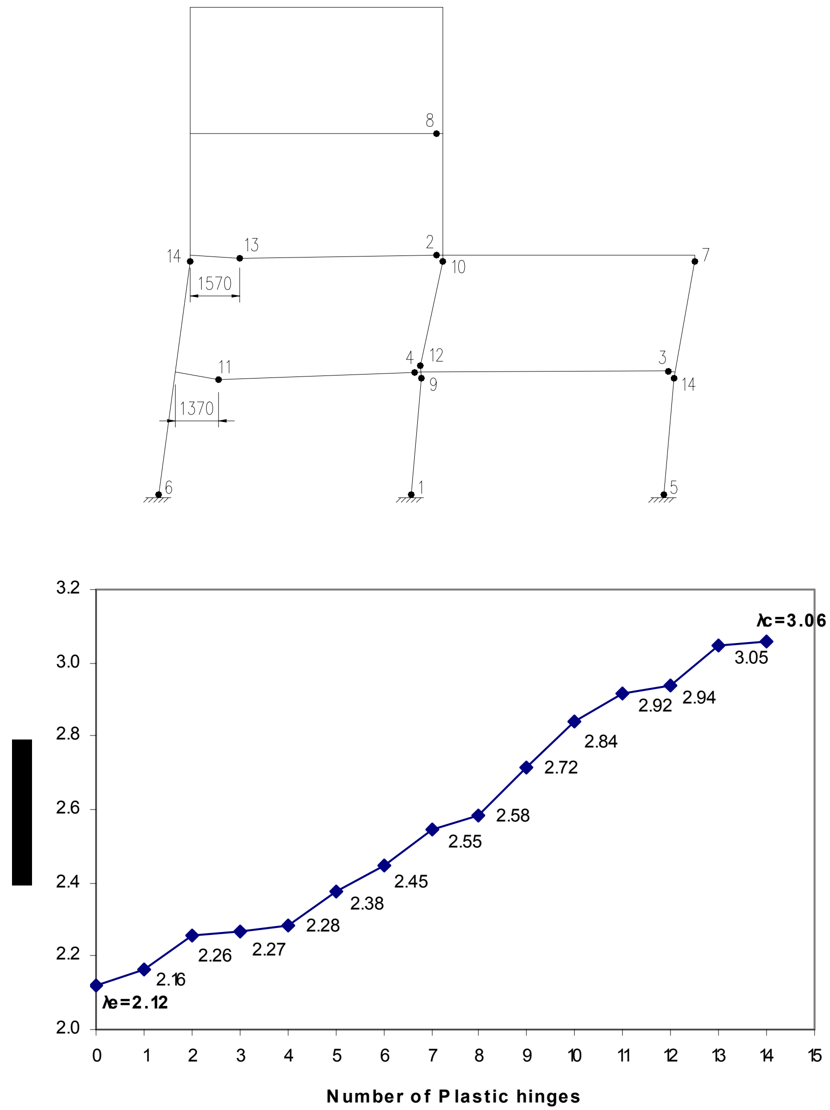


Figure 3.6 Collapse Mechanism and Load Factor λ in Case 5

DISCUSSION

4.1 Reduction in Plastic Moment Capacity due to Axial Force Effect

Regardless the reduction effects, the column's plastic moment is $M_p = Z_x F_y = 315.35kNm$. The maximum axial load that the section can carry, ignoring buckling, is $N_p = AF_y = 2992.5kN$. In order to investigate the axial force effect on the plastic moment capacity, one of the first floor columns in all cases, "column 1", are checked (the location of "column 1" is indicated in figure 3.1). Table 4.1 presents the analysis result of axial force N and reduced actual plastic moment M of the column at the collapse mechanism.

Case No.	1	2	3	4	5
$N(kN)$	309.7	493.7	549.5	631.4	374.9
$M_p(kNm)$	298.5	288.8	286.1	279.5	295.0
N/N_p	0.103	0.165	0.184	0.211	0.125
M/M_p	0.947	0.916	0.907	0.886	0.935

Table 4.1 Axial Force N and Plastic Moment M in "Column 1"

It is clear that the bigger axial force, the bigger reduction in plastic moment capacity. The reduction is not really too much of a problem in single-story frame, but can be much more so in multistory frames.

4.2 Comparison of Linear Analysis and Second-Order Analysis in Case 1, Case 3

The second-order analysis is done for Case 1 and Case 3. The analysis results are presented in figure 3.2, figure 3.4 and Table 4.2. The subscript 1 and 2 indicate the linear analysis and 2nd-order analysis result respectively.

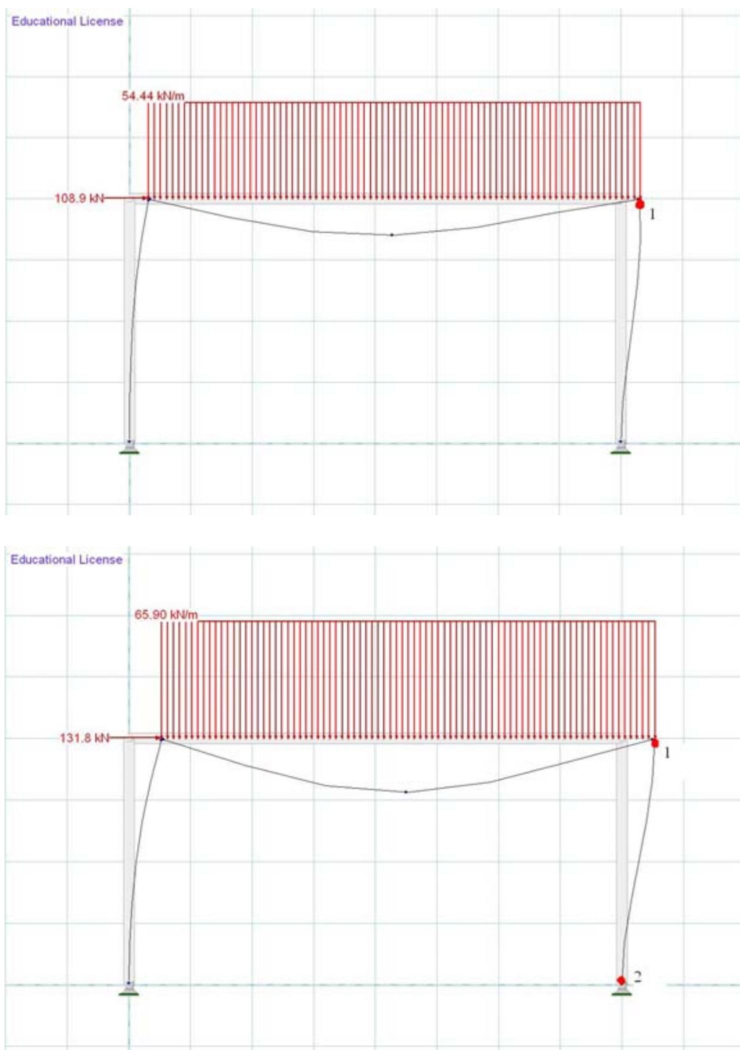
Case No.	λ_{c1}	λ_{c2}	$\lambda_{c2}/\lambda_{c1}$
1	7.70	7.45	0.97
3	1.95	1.82	0.93

Table 4.2 Collapse Load Factor λ_c in Linear Analysis and 2nd-Order Analysis

The analysis results show that the collapse load factor will be reduced when the load is considered acting on the deformed frame. The collapse load factor λ_c reduces 7% in Case 3 while 3% reduction in Case 1. So the $P-\Delta$ effect is more significant in multistory frame than in single-story frame.

4.3 Dr. Frame

Dr. Frame is a very handy tool to do the plastic analysis. It provides the ability to model plastic behavior in structure interactively. The procedure of the plastic hinges formation can be observed step by step. One can get the order of plastic hinges formation and the final collapse mechanism easily and visually. Figure 4.1 illustrates the procedure of plastic hinges formation in Case 1.



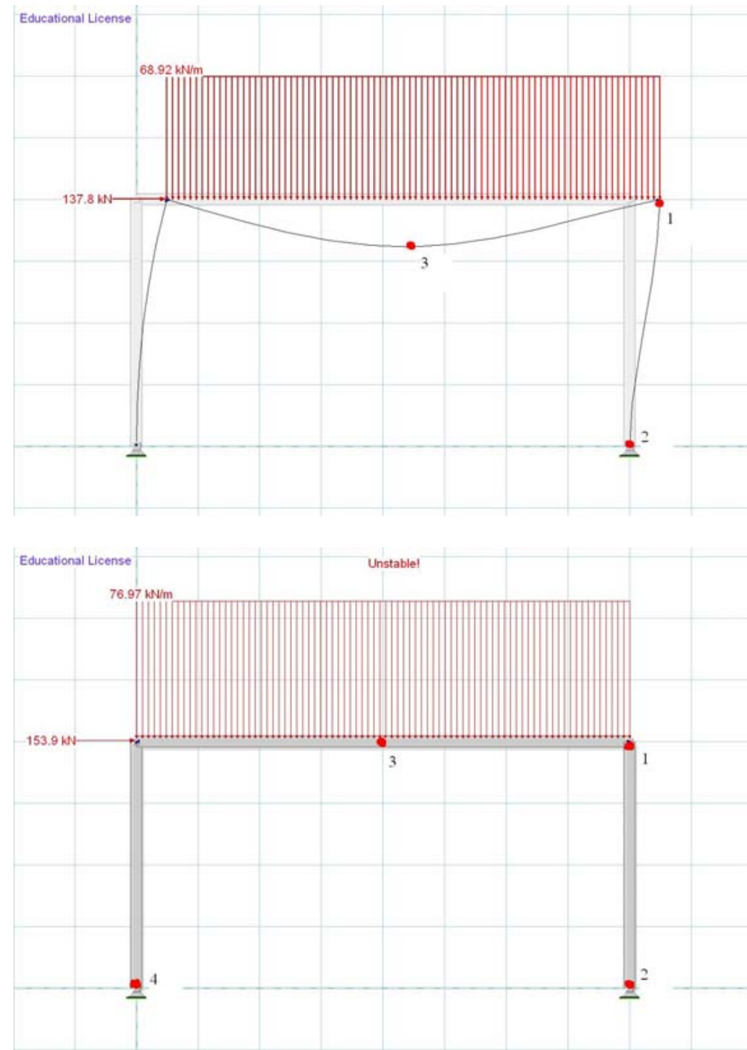


Figure 4.1 Procedure of Plastic Hinges Formation in Case 1

CONCLUSION

1. The plastic moment reduction due to axial force effect must be considered in the plastic analysis, especially when the axial force N exceeds $15\%N_p$.

2. A reasonable result can be got from the linear analysis. But for more accurate result, one needs to do the second-order analysis accounting for the $P-\Delta$ effects and load-dependent EI, especially for multistory frame.

3. For a framed structure of complicated shape and with a high degree of redundancy, it is not practical to discuss all possible collapse mechanisms. However, a perfect plastic analysis can be performed with the aid of advanced computer program, such as Dr. Frame.

4. The location of plastic hinges in frames to form a mechanism varies with the type of loading, as well as with shape and other physical properties of the frame.

REFERENCES

1. C. E. Massonnet and M. A. Save, Plastic Analysis and Design, Vol.1 Beams and Frames, (Blaisdell, 1965).

2. Stuart S. J. MOY, Plastic Methods for Steel and Concrete Structures, 2nd edition, (Macmillan, 1996).

3. T. J. Macginley, Steel Structures, Practical Design Studies, 2nd edition, (E & FN SPON, 1997).

4. Handbook of Steel Construction, 7th edition, (CISC, 2000).

5. Dr. Frame Users' Guide.

6. Civil 510 Lecture Notes.