

COMPARISON OF MEMBER DESIGN BETWEEN CSA & LRFD

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Abstract

Load and resistance factor design in accordance to CSA and LRFD, is based on limit states design which is, a rational approach to the design of structural steel for buildings. Although both codes are influenced by the same design method, there still exist some minor differences in the design procedures and limitations of structural members such as beams, column, etc. This report focuses on examining and comparing the design procedures presented in these two steel design handbooks by means of examples, spreadsheets and comparison graphs.

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1.0 Introduction

1.1 Limit States Design

Limit States Design, used in CSA and LRFD, is a design method in which the performance of a structure is checked to prevent with sufficiently small probability occurrence of various types of collapse and unserviceability.

There are 2 types of Limit States Design: Serviceability Limit States Design and Ultimate Limit State Design.

In Serviceability Limit States the behavior of the structure under normal operating conditions is examined. The types of structural behavior that may impair serviceability are:

- Excessive deflection or rotation
- Excessive vibration induced by wind or transient live loads

As a result, the design of the structure shall be based on the principle that no applicable strength or serviceability limit states shall be exceeded when the structure is subjected to applicable load conditions, and the designer attempts to ensure that the structure will fulfill its function satisfactorily when subjected to its service loads.

Ultimate Limit States, also defined as limit states of strength, defines safety against the extreme loads which may eventually result in overturning, sliding, fracture or collapse due to fatigue of other causes during the intended life of the structure. As a result, the structure must retain its load carrying capacity up to the factored load levels. In essence, the designer ensures that the maximum strength of a structure is greater that the imposed loads with a reasonable margin against failure.

CSA and LRFD, like other structural codes, focus on the limit states of strength because of overriding considerations of public

safety for the life, and property of human beings. Table 1.1, illustrates the Safety Index, β , the measure of probability of failure for various degrees of damage for both Serviceability and Ultimate Limit States. According to the table, the safety index values are much higher for Ultimate Limit States as there are for Serviceability Limit States, due to the fact that the latter deals with collapse and more severe case whereas the later examines functional performance and economy of design under operating service loads.

	Small	Medium	Severe
Ultimate	4.2	4.7	5.2
Serviceability	2.0	2.5	3.0

Table1.1- Safety Index, β

1.2 Limit States Design versus Allowable Stress Design

As mentioned earlier, Limit States Design checks the performance of the structure against various types of collapse or unserviceability. On the other hand, Allowable Stress design ensures that the stresses developed in a structure due to service loads do not exceed the elastic limit. Figure 1.1, compares these 2 design methods by plotting safety index, β , versus the length for a beam and a column under axial compression load. As shown in the upper part of the figure, CSA S16.1, which is based on limit states, gives more uniform safety as does CSA S16 which is based on Allowable Stress Design.

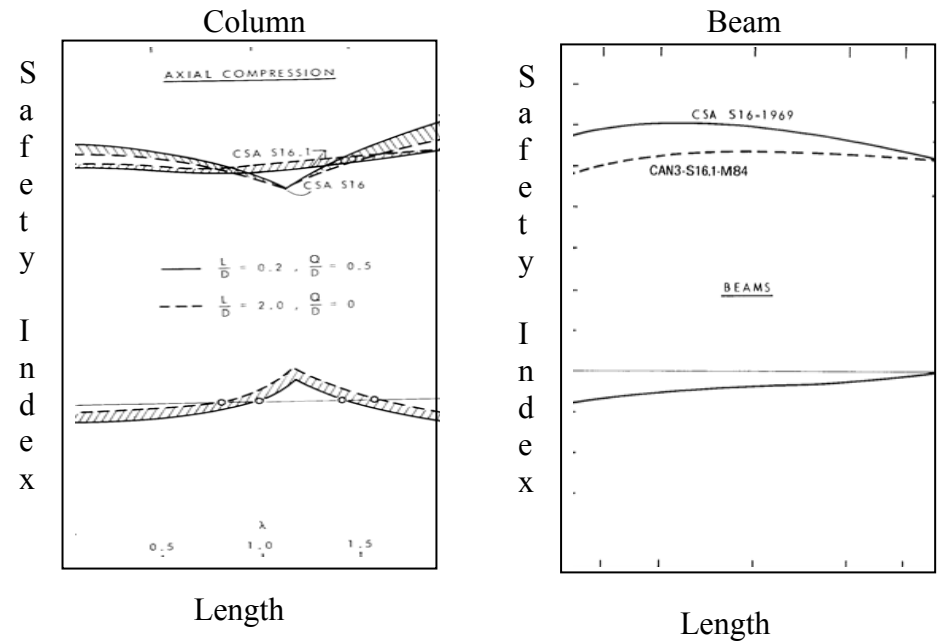


Figure 1.1- Limit States versus Allowable Stress Comparison Graph

1.3 Design Strength

Figure 1.2, shows a theoretical frequency distribution curve for the effect of factored load and factored resistance on structural elements. When the 2 curves overlap, as shown by shaded area, the loads acting on a structural element will exceed the resistance of the structural element and failure occurs.

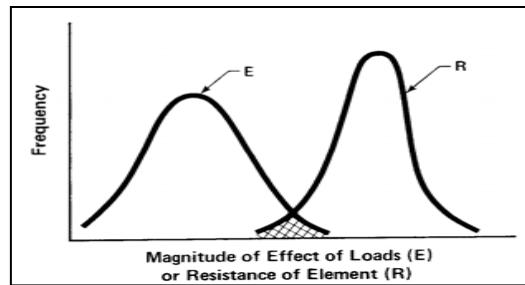


Figure 1.2-Frequency Distribution Curve

As a result, the structure shall be proportioned so that the overlap of the 2 curves is small and hence the probability of failure is small enough to be acceptable.

Since the loads acting on a structure and the resistance of the structural elements can only be defined statistically, the factor of safety is included in load and resistance calculations in Limit States Design. The load factor, applied to the specified load, takes into account the fact that the loads acting on the structural elements may be higher than anticipated and the resistance factor applied to the theoretical member strengths, takes into account of the fact that the resistance of the structural elements may be less than anticipated due to variation of materials, dimensions, etc.

The resulting design criteria in CSA and LRFD ensures that the loads acting on a structure shall always be less than the resistance of the structural elements by selecting the load and resistance factors and nominal load and resistance values which will never be exceeded under the design assumption.

$$\text{Factored Load} \leq \text{Effect of Factored resistance} \quad (1-1)$$

2.0 Tension Members

Tension members are structural elements that are subjected to direct axial loads, which tend to elongate the member. Typical examples of tension members can be found in hangers or cables supporting a floor or roof (Fig 2.1a), components of trusses (Fig 2.1b), tie-rods (Fig 2.1 c) and bracing systems (Fig 2.1 d).



(a)



(b)



(c)



(d)

Figure 2.1- Types of Tension Members

2.1 Design of Tension Members

The basic requirement for the design of tension member according to Limit States Design is to provide enough cross sectional area such that the factored resistance of the member exceeds the factored load.

There are 2 types of tensile resistance:

- Tensile yielding strength or unrestricted plastic flow of the cross section when deformation at yield is excessive. This type of strength represents the limit states in which the failure is gradual and as a result, the safety index of 3.0 is considered acceptable. CSA and LRFD give the same equation for the tensile yielding strength, taking the product of gross sectional area, A_g , and the yielding strength of material, F_y .

LRFD: $P_n = \theta t A_g F_y$ $\theta = 0.9$ (2-1)

CSA: $T_r = \theta A_g F_y$ $\theta = 0.9$ (2-2)

- Fracture of the net section at ultimate load occurs when there is sufficient ductility to provide reasonably uniform stress- distribution. The failure due to this fracture is sudden and with little warning. This has an effect of increase in the safety index from 3.0 to 4.5. The following equations are used for the fracture of the net section:

LRFD: $P_n = 0.75 A_e F_u$ (2-3)

CSA: $T_r = 0.85 \theta A_n F_u$ (2-4)

$T_r = 0.85 \theta A'_{ne} F_u$ (2-5)

Since there is no reserve of any kind beyond the ultimate resistance, an additional 0.85 multiplier is included in equations 2-4 and 2-5, results in the net resistance factor of 0.765 while LRFD uses 0.75 as resistance factor.

2.2 Tension Member Examples

2.2.1 Example 1-Bolted Connection

The first tension member example as shown in Figure 2.2 is the lower cord of a truss consisting of 2 C310x45 sections tied across the flanges with lacing bars. Both the flanges and web plates are provided to transfer the stress from one section to the other.

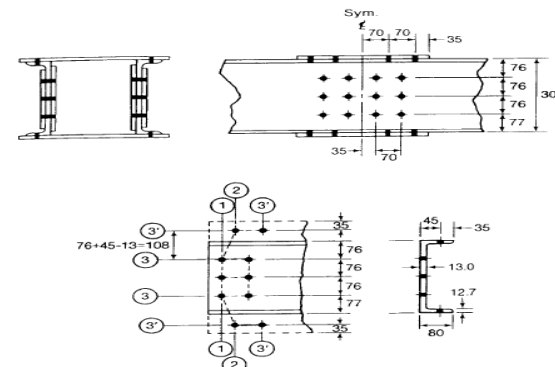


Figure 2.2- Example 1-Bolted Connection

As shown in the figure, all parts of the member (both flanges and web) are connected by bolts. In this case, the effective net area is the sum of the net areas:

$A_e = A_n = \sum (W_g - \sum d_n + S^2/4g) \times t$ (2-6)

The first 2 terms represent the net area of the section that is perpendicular to the force and is in direct tension (section 1-1 in Figure 2.2), and the last term in the equation represents the net area of the segment that is inclined to the force (section 2-2 in Figure 2.2).

CSA				LRFD			
1 Cross Section Yield							
Tension Resistance_Yield	$T_{r,CSA} = 2 \cdot A_n \cdot F_u / 1000$	=	3585 kN	886 kip	$F_u \cdot A_n \cdot 2 / 9 \cdot A_g \cdot F_y$		
2 Fracture at Net Section							
Section 1-1							
w_g	$w_g = d - b - t$	=	439.0 mm	17.3 in			
A_{n1}	$A_{n1} = w_g \cdot t$	=	3610 mm ²	14.2 in ²	A_{n1}		
A_{n2}	$A_{n2} = 2 \cdot w_g \cdot t$	=	9080.0 mm ²	14.5 in ²	U_1 [Both Flange and web Connected]		
Tension Resistance 1	$T_{r1} = A_{n1} \cdot 0.85 \cdot F_u / 1000$	=	3447 kN	14.5 in ²	$A_{n1} \cdot A_n \cdot U_1$		
Section 2-2							
A_{n2}	$A_{n2} = [A_g \cdot (1 - (d_h)^2 / t^2) \cdot (e_{horz} / 2) + (e_{vert} + b - e_{horz}) \cdot t]$	=	8147.5 mm ²	12.6 in ²	$A_{n2} \cdot A_n \cdot U_2$		
Tension Resistance 2	$T_{r2} = A_{n2} \cdot 0.85 \cdot F_u / 1000$	=	2992 kN	659 kip	$F_n \cdot 0.75 \cdot A_{n2} \cdot F_u$		
Section 3-3							
A_{n3-1} (web)	$A_{n3-1} = [0.6 \cdot (s + e_{horz} - 1.5 \cdot d_h) \cdot t^2 + (g_1 + g_2 - 2 \cdot d_h) \cdot t^2]$	=	4659.2 mm ²	11.648 in ²	A_{n3-1}		
A_{n3-2} (Flanges)	$A_{n3-2} = [0.6 \cdot (s + e_{horz} - 1.5 \cdot d_h) \cdot t^2 + (e_{horz} - 5 \cdot d_h) \cdot t^2]$	=	4295.2 mm ²	10.738 in ²	A_{n3-2}		
A_{n3} Total	$A_{n3} = A_{n3-1} + A_{n3-2}$	=	8954.40 mm ²	22.39 in ²	A_{n3}		
Tension Resistance 3	$T_{r3} = A_{n3} \cdot 0.85 \cdot F_u / 1000$	=	3288 kN	1169 kip	$F_n \cdot 0.75 \cdot A_{n3} \cdot F_u$		
The least Tension Force	$T_r = \text{MIN}(T_{r1}, T_{r2}, T_{r3}, T_{r,CSA})$	=	2992 kN	659 kip			

Figure 2.3- Example 1-Calculation

Figure 2.3 shows the calculation of the tensile capacity of this connection in CSA and LRFD. According to the results, the tensile capacity is 2992 kN in CSA and is calculated to be 2933 kN in LRFD. The difference in the results is due to the difference in resistance factor applied in 2 codes. In other words, LRFD uses 0.75 as resistance factor whereas CSA uses 0.85 or 0.765 as resistance factor.

2.2.2 Example 2- Shear Lag in Bolted Connection

The second example is a C380x50 channel which is only connected through web. As a result, shear lag phenomena is occurring in this case due to the fact that the connected part (in this

case the web) tends to reach ultimate strength before the net section strength is reached.

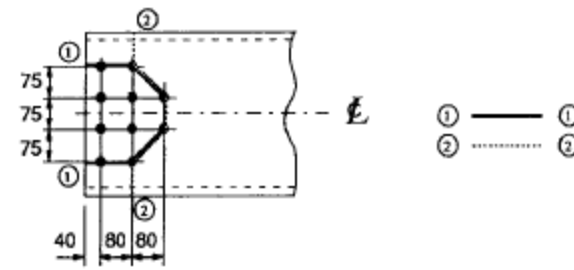


Figure 2.4- Example 2-Shear Lag in Bolted Connection

As a result, in order to take into account of this phenomena, a shear lag reduction factor, U, is multiplied to the effective net area. The calculation steps and results in both codes are illustrated in Fig 2.5.

CSA				LRFD			
1 Checking Bolt Shear:							
Connection Bolt Shear	$V_{CSA} = \sum V_i$	=	878 kN	878 kN	$\phi \cdot V_n$		850.0 OK
2 Cross Section Yield							
Tension Resistance_Yield	$T_{r,CSA} = 2 \cdot A_n \cdot F_u / 1000$	=	3585 kN	886 kip	$F_u \cdot A_n \cdot 2 / 9 \cdot A_g \cdot F_y$		850.0 OK
3 Fracture at Net Section							
Section 1-1							
w_g	$w_g = d - b$	=	439.0 mm	17.3 in			
A_{n1}	$A_{n1} = w_g \cdot t$	=	499.0 mm ²	0.8 in ²	A_{n1}		
A_{n2}	$A_{n2} = 2 \cdot (w_g + 1.5 \cdot d_h) \cdot t$	=	908.0 mm ²	6.4 in ²	A_{n2}		
A_{n3}	$A_{n3} = 2 \cdot (t \cdot (s + 1.5 \cdot d_h))$	=	3914.0 mm ²	15 in ²	A_{n3}		
Total A_{n1}	$A_{n1} = A_{n1} + A_{n2} + A_{n3}$	=	2500.0 mm ²	4.6 in ²	A_{n1}		
Tension Resistance 1	$T_{r1} = A_{n1} \cdot 0.85 \cdot F_u / 1000$	=	1907.2 kN	444 kip	$A_{n1} \cdot A_n \cdot U_1$		
Section 2-2							
A_{n2}	$A_{n2} = [A_g \cdot (1 - (d_h)^2 / t^2) \cdot (e_{horz} / 2) + (e_{vert} + b - e_{horz}) \cdot t]$	=	5694.4 mm ²	9.0 in ²	A_{n2}		
Calculating Shear Lag	$U = 0.85 \cdot A_{n2} / A_{n1}$	=	4933.7 mm ²	6.3 in ²	U_1 [length of connection]		
Tension Resistance 2	$T_{r2} = A_{n2} \cdot 0.85 \cdot F_u / 1000$	=	1698.4 kN	395 kip	$A_{n2} \cdot A_n \cdot U_2$		
The least Tension	$T_r = \text{MIN}(V_{CSA}, T_{r1}, T_{r2})$	=	878 kN	197 kip	$F_n \cdot 0.75 \cdot A_{n2} \cdot F_u$		850.0 OK
Adequacy of Connection			Connection Design ok	878 kN			191.1 OK

Figure 2.5- Example 2-Calculation

As also shown in Figure 2.5, the results according to both codes turn out to be the same (878 kN), since the tensile capacity of connection is governed by the tensile yielding strength in both cases.

2.2.3 Example 3- Welded Connection

The final tensile strength example as shown in Fig 2.6 is a Hollow Structural Section which is connected in the end by a fillet weld to 1/2 in. thick single concentric gusset plate.

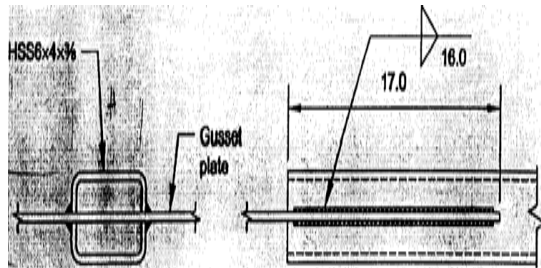


Figure 2.6- Example 3-Welded Connection

The effective net area is the product of the net area and the shear lag reduction factor. In this example, the tensile capacity of the connection is governed by the fracture of the net section.

CSA		LRFD				
Factored Tension Force	$T_f = 1.25 \cdot DL + 1.5 \cdot LL$	=	895.5 kN	$1.2 \cdot DL + 1.6 \cdot LL$	=	240.0 kip
Net Area	$A_n = A_g - 2 \cdot (t_p + 2) \cdot t$	=	3954.1 mm ²		=	6.1 mm ²
	$U = 1/(nL)$	=	0.90		=	0.9
Net Effective Area	$A_{ne} = U \cdot A_n$	=	3557.52 mm ²		=	5.5 mm ²
	ϕ	=	0.9		=	0.9
Tension Resistance	$T_r = \phi \cdot 0.85 \cdot A_{ne} \cdot F_u / 1000$	=	1088.6 kN	$A_{ne} \cdot F_u \cdot 0.75$	=	239.9 kip
	$\text{if}(T_r > T_f, \text{"Design OK"}, \text{"Design Not OK"})$	=	Design OK		=	Design OK

Figure 2.7- Example 3-Calculation

As shown in Figure 2.7, the tensile capacity of the connection is 1089 kN according to CSA and 1067 kN according to LRFD. Once again, the difference in results is due to the difference in resistance factors used in 2 codes.

3.0 Compression Members

Compression members are subjected to loads that tend to decrease the length.



Fig 3.1- Steel Column

3.1 Design of Compression Members

Compression members need to be checked for limit states flexural buckling and torsional or flexural torsional buckling. Moreover, the cross section of the member shall also be checked for local buckling to ensure that the cross section does not buckle before the member fails as a unit.

3.1.1 Compressive Strength for Flexural Buckling

The following equations are the flexural buckling equations for compression members.

CSA: $C_r = \theta A F_y (1 + \lambda^{2n})^{-1/n} = (0.9) A F_y (1 + \lambda^{2n})^{-1/n}$ (3-1)

LRFD: $\phi_c P_n = (0.85) A_g F_{cr}$ (3-2)

Inelastic Buckling $\lambda_c \leq 1.5$ $F_{cr} = (0.658^{\lambda_c^2}) F_y$ (3-3)

Elastic Buckling $\lambda_c > 1.5$ $F_{cr} = (0.877/\lambda_c^2) F_y$ (3-4)

$\lambda = \lambda_c = (kL/r\pi) (F_y/E)^{1/2} = (F_y/F_e)^{1/2}$ (3-5)

The equations turn out to give almost same result. The only difference between the codes in flexural buckling is the difference in resistance factors. As shown in equations 3-1, CSA uses 0.9 as resistance factor while LRFD uses 0.85 as resistance factor in flexural buckling equations (equation 3-2).

3.1.2 Compressive Strength for Flexural-Torsional Buckling

The following equations are used to calculate the flexural-torsional buckling of a compression member. Both CSA and LRFD use the same equations for calculating the critical stress, F_e . However, similar to flexural buckling, CSA uses 0.9 as resistance factor while LRFD uses the resistance factor of 0.85 in calculating the resulting compressive strength due to flexural torsional buckling (equations 3-1 & 3-2).

CSA & LRFD:

Doubly Symmetric and Axisymmetric sections, F_e is the least of

$F_{ex} = \pi^2 E / (k_x L_x / r_x)^2$ (3-6)

$F_{ey} = \pi^2 E / (k_y L_y / r_y)^2$ (3-7)

$F_{ez} = (\pi^2 E C_w / (k_z L_z)^2 + GJ) (1/A r_0^2)$ (3-8)

Singly Symmetric Sections with y for axis of symmetry, F_e is the lesser of F_{ey} and F_{ez}

$F_{eyz} = F_{ey} + F_{ez} / (2\Omega) [1 - (1 - (4F_{ey}F_{ez}\Omega) / (F_{ey} + F_{ez})^2)^{1/2}]$ (3-9)

Asymmetric Sections, F_e is the smallest root of

$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2 (F_e - F_{ey}) (x_0/r_0)^2 - F_e^2 (F_e - F_{ex}) (y_0/r_0)^2 = 0$ (3-10)

$r_0^2 = x_0^2 + y_0^2 + r_x^2 + r_y^2$ (3-11)

$\Omega = 1 - [(x_0^2 + y_0^2) / r_0^2]$ (3-12)

3.1.3 Compressive Strength for Local Buckling

Table B4.1 of LRFD and Table 1 of CSA show the limiting values of axial members for various sections. The difference between the codes in local buckling calculations is in slender compressive elements. LRFD introduces a non-dimensional reduction factor Q_s in the local buckling equations of unstiffened compression elements whose width-to-thickness ratio exceeds the applicable non-compact limit (λ_r). The resulting equations for calculating the local buckling on these elements are shown below.

LRFD:

Single Angles

$b/t \leq 0.446 (E/F_y)^{1/2}$ $Q_s = 1.0$ (3-13)

$$0.446 (E/F_y)^{1/2} < b/t < 0.91 (E/F_y)^{1/2} \quad Q_s = 1.34 - 0.76(b/t) (E/F_y)^{1/2} \quad (3-14)$$

$$\text{If } b/t \geq 0.91 (E/F_y)^{1/2} \quad Q_s = (0.534E) / (F_y (b/t)^2) \quad (3-15)$$

Flanges, angles and plates projecting from rolled beams or columns or other compression members

$$95 / (F_y)^{1/2} < b/t < 155 / (F_y)^{1/2} \quad Q_s = 1.415 - 0.00437(b/t) (F_y)^{1/2} \quad (3-16)$$

$$b/t \geq 176 / (F_y)^{1/2} \quad Q_s = 20000 / [F_y (b/t)^2] \quad (3-17)$$

Flanges, angles and plates projecting from built-up columns or other compression members

$$109 / (F_y/k_c)^{1/2} < b/t < 200 / (F_y/k_c)^{1/2} \quad Q_s = 1.415 - 0.00437(b/t) (F_y)^{1/2} \quad (3-18)$$

$$b/t \geq 200 / (F_y/k_c)^{1/2} \quad Q_s = 26200k_c / [F_y (b/t)^2] \quad (3-19)$$

where k_c is,

$$\text{I-shaped} \quad 4/(h/t_w)^{1/2}, 0.35 \leq k_c \leq 0.763 \quad (3-20)$$

$$\text{Other} \quad 0.763$$

Stems of tees

$$127 / (F_y)^{1/2} < b/t < 176 / (F_y)^{1/2} \quad Q_s = 1.908 - 0.00715(b/t) (F_y)^{1/2} \quad (3-21)$$

$$b/t \geq 176 / (F_y)^{1/2} \quad Q_s = 20000 / [F_y (b/t)^2] \quad (3-22)$$

For slender stiffened compression elements, a reduced effective width b_e shall be used instead of actual b in calculating the design properties of the section containing the element. The following equations are used to calculate the reduced effective width, b_e , for various sections:

For flanges of square and rectangular sections of uniform thickness

$$b/t \geq 23 / (f)^{1/2} \quad b_e = 326t / (f)^{1/2} [1 - 64.9 / (b/t)(f)^{1/2}] \quad (3-23)$$

Otherwise $b_e = b$

Other uniformly compressed elements

$$b/t \geq 253 / (f)^{1/2} \quad b_e = 326t / (f)^{1/2} [1 - 57.2 / (b/t)(f)^{1/2}] \quad (3-24)$$

Otherwise $b_e = b$

Q in all these cases is equal to Q_s

Axially loaded circular sections with diameter-to-thickness ratio D/t greater than $3300/F_y$ but less than $13000/F_y$

$$Q = Q_a = 1100 / F_y (D/t) + (2/3) \quad (3-25)$$

$$\text{Where } Q_a = (\text{effective area}) / (\text{actual area}) \quad (3-26)$$

The critical stress F_{cr} , is

$$\lambda_c(Q)^{1/2} \leq 1.5 \quad F_{cr} = Q (0.658 Q^{\lambda_c^2}) F_y \quad (3-27)$$

$$\lambda_c(Q)^{1/2} > 1.5 \quad F_{cr} = Q(0.877 / \lambda_c^2) F_y \quad (3-28)$$

$$Q = Q_a Q_s \quad (3-29)$$

Cross sections comprised of unstiffened elements, $Q = Q_s$, $Q_a = 1.0$

Cross sections comprised of stiffened elements, $Q = Q_a$, $Q_s = 1.0$

Cross sections comprised of both stiffened and unstiffened elements, $Q = Q_a Q_s$

CSA, on the other hand, uses an effective area using reduced element widths meeting the maximum width-to-thickness ratio of a class 3 or an effective yield stress determined from the width-to-thickness ratio meeting the class 3 limit for calculation of compressive resistance, C_r .

4.0 Flexural Members

Members subject to simple bending are loaded in a plane parallel to a principal axis that passes through the shear center or is restrained against twisting at load points and support. The following are various types of flexural members:

- Doubly symmetric compact I-shaped members and channels bent about their major axis
- Doubly symmetric I-shaped members with non-compact or slender flanges and compact webs
- Doubly Symmetric I-shaped members with non-compact webs
- Singly symmetric I-shaped members with compact or non-compact webs

- Doubly symmetric I-shaped members with slender webs (Plate Girder) bent about major axis
- I-shaped members and channels bent about major axis
- Square and rectangular HSS and box-shaped members
- Round HSS and pipes
- Tees and double angles
- Single angles
- Rectangular bars and rounds
- Unsymmetrical shapes

In order to better compare each code approach in design of flexural members, 2 of the above sections were chosen and the values of moment were computed for various lengths according to each code. In the end, moments were plotted for various lengths for each code in both examples and the resulting graphs were compared.

4.1 Design of Flexural Members Examples

The following examples compare the design of members for bending between CSA and LRFD.

4.1.1 Example1-Doubly Symmetric Compact I-shaped Member Bent about Major Axis

The following equations were used to calculate the moment for different lengths.

LRFD:

$$\text{Yielding} \quad \phi_b M_n = \phi_b M_p = \phi_b F_y Z_x \quad (4-1)$$

Lateral Torsional Buckling

If $L_b \leq L_p$ Limit states lateral torsional buckling does not apply

If $L_p < L_b \leq L_r$

$$\theta_b M_n = \theta_b C_b [M_p - (M_p - 0.7 S_x F_y) ((L_b - L_p) / (L_r - L_p))] \leq \theta M_p \tag{4-2}$$

If $L_b > L_r$ $\theta_b M_n = \theta_b (\pi / L_b) (EI_y GJ + (\pi E / L_b)^2 I_y C_w)^{1/2}$ (4-3)

where,

$$L_p = 300 r_y / (F_y)^{1/2} \tag{4-4}$$

$$L_r = (r_y X_1) / (F_y - F_r) (1 + (1 + X_2 (F_y - F_r)^2)^{1/2})^{1/2} \tag{4-5}$$

$$X_1 = (\pi / S_x) (EGJ A^2)^{1/2} \tag{4-6}$$

$$X_2 = (4 C_w / I_y) (S_x / GJ)^2 \tag{4-7}$$

CSA:

$M_u > 0.67 M_p$ $M_r = 1.15 \theta M_p (1 - (0.28 M_p / M_u)) \leq \theta M_p$ (4-8)

$M_u \leq 0.67 M_p$ $M_r = \theta M_u$ (4-9)

where,

$$M_u = w_2 \pi / L (EI_y GJ + (\pi E / L)^2 I_y C_w)^{1/2} \tag{4-10}$$

$$w_2 = 1.75 + 1.05k + 0.3k^2 \leq 2.5 \tag{4-11}$$

$$k = (\text{small factored moment}) / (\text{larger factored moment}) \tag{4-12}$$

Class 1 or 2 Doubly Symmetric I-shaped member bending about the major axis:									
Example :		W24x68		Mfx	107				
				Mfy	67				
						kN.m			
						kN.m			
Unsupported Length	CSA			LRFD			Biaxial Bending Ratio	Moment Difference	
	Mrx	Mry	Biaxial Bending Ratio	Mnx	Mny	Mnx			Mny
(mm)	kN.m	kN.m	Mrx/Mrx + Mfy/Mry	kip.ft	kip.ft	kN.m	kN.m	Mfx/Mfx + Mfy/Mfy	
1000	914	127	0.64	664	92	885	123	0.66	3.1
1200	914	127	0.64	664	92	885	123	0.66	3.1
1400	914	127	0.64	664	92	885	123	0.66	3.1
1600	914	127	0.64	664	92	885	123	0.66	3.1
1800	914	127	0.64	664	92	885	123	0.66	3.1
2000	914	127	0.64	664	92	885	123	0.66	3.1
2200	914	127	0.64	652	92	869	123	0.67	4.9
2400	914	127	0.64	640	92	853	123	0.67	6.6
2600	914	127	0.64	628	92	837	123	0.67	8.4
2800	914	127	0.64	616	92	821	123	0.67	10.1
3000	901	127	0.65	603	92	804	123	0.68	10.8
3200	882	127	0.65	591	92	788	123	0.68	10.7
3400	862	127	0.65	579	92	772	123	0.68	10.4
3600	840	127	0.65	567	92	756	123	0.69	10.0
3800	818	127	0.66	555	92	740	123	0.69	9.5
4000	795	127	0.66	542	92	723	123	0.69	9.1
4200	771	127	0.67	530	92	707	123	0.7	8.3
4400	746	127	0.67	518	92	691	123	0.7	7.4
4600	721	127	0.67	506	92	675	123	0.7	6.4

Fig 4.1- Moment vs. Length (W610x101)

In Figure 4.2, the moment versus the length is plotted for both CSA and LRFD.

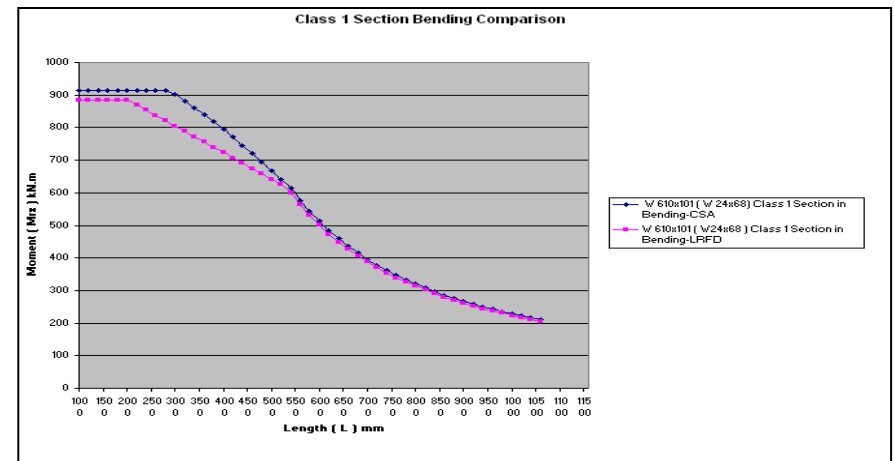


Figure 4.2- Plot of Moment vs. Length for W610x101

According to Figure 4.2, the upper and the lower curves illustrate the moment variation versus length according to CSA and LRFD respectively. As shown in the figure, the curves have almost the same shape and at lengths less than compact limit length, L_p , have moment difference of about 3%. However, at length between compact and non-compact limit length ($L_p < L_b \leq L_r$), the difference between moment values tend to increase up to almost 11% (length 3000mm). Finally at length of 5400 mm where the length of the member exceeds non-compact limit length, ($L_b > L_r$) the curves tend to coincide.

4.1.2 Example2- Rectangular Hollow Structural Section

The following equations are used for calculating the moment of Rectangular Hollow Structural Sections:

LRFD:

Yielding

$$\theta b M_n = \theta b F_y Z \tag{4-13}$$

Flange local buckling

Compact sections: does not apply

Non-compact sections:

$$M_n = M_p - (M_p - F_y S_x)(3.57(b/t)(F_y/E)^{1/2} - 4.0) \leq M_p \tag{4-14}$$

Web local buckling

Compact sections: does not apply

Non-compact webs

$$M_n = M_p - (M_p - F_y S_x)(0.305(h/t_w)(F_y/E)^{1/2} - 0.738) \leq M_p \tag{4-15}$$

CSA:

Class 1 & 2 sections: $M_r = \theta Z F_y = \theta M_p \tag{4-16}$

Class 3 sections: $M_r = \theta S F_y = \theta M_y \tag{4-17}$

Figure 4.3 illustrates the moment values for various lengths for HSS 254x152x13 .

Laterally Unsupported beams:

Class 1 Hollow Structural Section									
Example :	HSS 254x152x13			M _{fx}	93	kN.m			
				M _{fy}	53	kN.m			
Unsupported Length	CSA			LRFD					
	M _x kLm	M _y kLm	Biaxial Bending Ratio M _{fx} M _{rx} + M _{fy} M _{ry}	M _x kip.ft	M _y kip.ft	M _x kLm	M _y kLm	Biaxial Bending Ratio M _{fx} M _{rx} + M _{fy} M _{ry}	Difference
1000	235	164	0.86	164	115	219	153	0.78	6.81
1200	235	164	0.86	164	115	219	153	0.78	6.81
1400	235	164	0.86	164	115	219	153	0.78	6.81
1600	235	164	0.86	164	115	219	153	0.78	6.81
1800	235	164	0.86	164	115	219	153	0.78	6.81
2000	235	164	0.86	164	115	219	153	0.78	6.81
2200	235	164	0.86	164	115	219	153	0.78	6.81
2400	235	164	0.86	164	115	219	153	0.78	6.81
2600	235	164	0.86	164	115	219	153	0.78	6.81
2800	235	164	0.86	164	115	219	153	0.78	6.81
3000	235	164	0.86	164	115	219	153	0.78	6.81
3200	235	164	0.86	164	115	219	153	0.78	6.81
3400	235	164	0.86	164	115	219	153	0.78	6.81
3600	235	164	0.86	164	115	219	153	0.78	6.81
3800	235	164	0.86	164	115	219	153	0.78	6.81
4000	235	164	0.86	164	115	219	153	0.78	6.81
4200	235	164	0.86	164	115	219	153	0.78	6.81

Fig 4.3- Moment vs. Length (HSS 254x152x13)

As shown in the above table, the calculated moments are different by 6.81%. The resulting moment values according to LRFD and CSA are plotted in Figure 4.4.

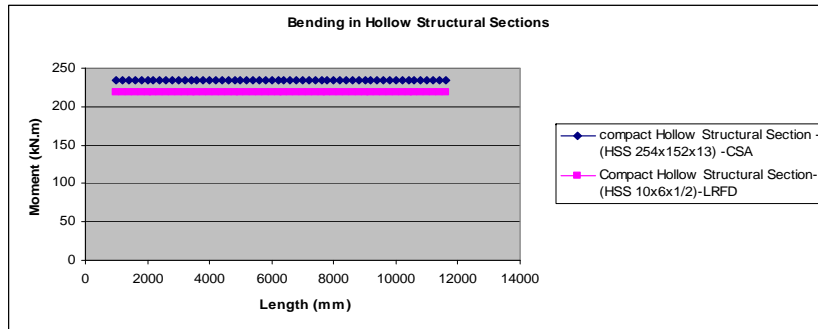


Figure 4.4- Plot of Moment vs. Length for HSS 254x152x13

Since in both design codes the design moment is the plastic moment, the difference in results is due to the difference in yield stress of steel. In other words, CSA uses 350 MPa for yield stress whereas LRFD uses 50 ksi (345 MPa) for yield stress of steel.

5.0 Beam-Columns

Beam columns refer to structural elements that are subjected to both axial load and moment hence the structural behavior need to be checked for the combination of two.

5.1 Design of Beam Column

LRFD uses 2 equations to check the capacity of a beam-column based on the ratio of factored axial load to factored axial resistance of the member.

$$Pr/P_c < 0.2 \quad Pr / (2P_c) + (M_{rx}/M_{cx}) + (M_{ry}/M_{cy}) \leq 1.0 \quad (4-17)$$

$$Pr/P_c \geq 0.2 \quad Pr/P_c + (8/9) (M_{rx}/M_{cx} + M_{ry}/M_{cy}) \leq 1.0 \quad (4-18)$$

CSA, on the other hand, uses a different method to examine the capacity of a beam-column. For class 1 & 2 sections of I-shaped members the following equation is used:

$$C_f/C_r + 0.85U_1xM_{fx}/M_{rx} + \beta U_1yM_{fy}/M_{ry} \leq 1.0 \quad (4-19)$$

$$\beta = 0.6$$

For all classes of sections the following equation is used:

$$C_f/C_r + U_1xM_{fx}/M_{rx} + U_1yM_{fy}/M_{ry} \leq 1.0 \quad (4-20)$$

In both cases, the member strength and stability need to be checked for cross-sectional strength, overall member strength, lateral-torsional buckling and finally bi-axial bending.

The following example is provided to better illustrate each code's approach in checking capacity of beam-columns. The first example shows a sample beam-column at specified length of 2000 mm under specified axial load and moments. In the second example, the length of the member has been increased to 8000 mm and the results were examined accordingly. The following 2 sections illustrate these examples' calculation process and results according to each code.

5.1.1 Example1- I-Shaped Compact Section at Length of 2000 mm

This example shows the behavior of a W24x68 beam column when is subjected to the following loads:

Cf=500 kN
Mfx=150 kN.m
Mfy=100 kN.m
Lx=Ly=2000 mm

LRFD:

Section Class:

flange: $bf/2tf = (8.97 / (2 \times 0.585)) = 7.67$
flange compactness $= 0.56(E/Fy)^{1/2} = 0.56(29000/50)^{1/2} = 13.48$
since $7.67 < 13.48 \rightarrow$ flange is compact
web: $h/t_w = 52$
web compactness $= 1.49(E/Fy)^{1/2} = 1.49(29000/50)^{1/2} = 11.14$
since $52 > 11.14 \rightarrow$ web is slender

Compression Strength:

$P_c = \theta c P_n = \theta c A_g F_{cr}$

Slenderness Ratio:

$kL_x/r_x = (1.0 \times 78.7 / 9.55) = 8.24$
 $kL_y/r_y = (1.0 \times 78.7 / 1.87) = 42.1$
max slenderness ratio = 42.1
 $\lambda_c = (kL_y/r_y \pi) (F_y/E)^{1/2} = (42.1/\pi) (501/29000)^{1/2} = 0.56$
since $\lambda_c < 1.5 \rightarrow F_{cr} = (0.658^{\lambda_c^2}) F_y = (0.658^{0.56^2}) 50 = 44.7$ ksi
 $P_c = (0.85) (20.10 \text{ in}^2) (44.7 \text{ ksi}) = 764$ kips
 $P_r/P_c = 112.5/764 = 0.15 < 0.2 \rightarrow$
 $P_r/2P_c + (M_{rx}/M_{cx}) + (M_{ry}/M_{cy}) \leq 1.0$

Moment:

Mcx:

$L_p = 300 r_y / F_y^{1/2} = (300 \times 1.87 / 50^{1/2}) = 78.6$ in
 $X_1 = (\pi / S_x) (EGJA/2)^{1/2} = (\pi / 154) (29000 \times 11200 \times 1.9 \times 20.10 / 2)^{1/2} = 1606.6$
 $X_2 = (4C_w / I_y) (S_x / GJ)^2 = (4 \times 9430 / 70.40) (154 / (11200 \times 1.9))^2 = 0.028$
 $L_r = (r_y X_1) / ((F_y - F_r) (1 + (1 + X_2 (F_y - F_r)^2)^{1/2})^{1/2}) = 206.3$ in
 $L_p < L_b < L_r \rightarrow$
 $M_{cx} = \theta b M_n = \theta b C_b [M_p - (M_p - 0.7 S_x F_y) ((L_b - L_p) / (L_r - L_p))] \leq \theta M_p$
 $= (677 - 473.6) ((78.7 - 78.6) / (206.3 - 78.6)) = 676.8$ kip.ft
 $M_{cy} = 0.9 Z_y F_y / 12 = (0.9) (24.5) (50) / 12 = 93.7$ kip.ft

Interaction Formula:

$P_r/2P_c + (M_{rx}/M_{cx}) + (M_{ry}/M_{cy}) = (112.5/764 \times 2) + (112.5/676.8) + (75/93.7) = 1.04 > 1.0$
Design not Ok!

CSA:

Section Class:

flange: $bf/2tf = (228 / (2 \times 14.9)) = 7.67$
since $7.67 < 145 / (F_y)^{1/2} = 7.8 \rightarrow$ flange is Class 1
web: $h/t_w = (603 - 2 \times 14.9) / 10.5 = 54.6$
since $52 < 1100 / (F_y)^{1/2} [1 - 0.39(C_f / \theta A F_y)] = 58.8 \rightarrow$ web is Class 1
Section is Class 1

Interaction Formula:

$C_f / C_r + 0.85 U_1 x M_{fx} / M_{rx} + \beta U_1 y M_{fy} / M_{ry} \leq 1.0$
 $\beta = 0.6$

a) Cross sectional strength:

$$C_r = \theta A_g F_y = (0.9) (13000) (350) = 4095 \text{ kN}$$

$$M_{rx} = \theta Z_x F_y = (0.9) (2.9 \times 10^6) (350) = 913.5 \text{ kN.m}$$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_{1x} = U_{1y} = 1.0$$

$$(500/4095) + (0.85 \times 1.0 \times 150/913.5) + (0.6 \times 1.0 \times 100/127.3) = 0.73 < 1.0$$

b) Overall member strength:

$$C_r = C_{r0} = \theta A_g F_y (1 + \lambda^{2n})^{(-1/n)}$$

$$k_{Lx}/r_x = (1.0 \times 2000/243) = 8.23$$

$$k_{Ly}/r_y = (1.0 \times 2000/47.7) = 41.9 \rightarrow \text{maximum slenderness ratio}$$

$$\lambda = \sqrt{(k_{Ly}/r_y)(F_y/\pi^2 E)} = (41.9)(350/200000 \times \pi^2)^{1/2} = 0.558$$

$$C_{r0} = (0.9) (13000 \times 350) (1 + (0.558)^{2 \times 1.34})^{-1/1.34} = 3553 \text{ kN}$$

$$M_{rx} = \theta Z_x F_y = (0.9) (2.9 \times 10^6) (350) = 913.5 \text{ kN.m}$$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_{1x} = [w_1 / (1 - (C_f/C_e))]$$

$$w_1 = 0.6 - 0.4k = > 0.4$$

$$k = (M_{fx\text{Small}}) / (M_{fx\text{Large}}) = 0/150 = 0$$

$$w_1 = 0.6$$

$$C_{ex} = \pi^2 E I_x / L_x^2 = (\pi^2 \times 200000 \times 764 \times 10^6 / 2000^2) = 377019$$

$$U_{1x} = 0.6 / (1 - 500/377019) = 0.6$$

$$U_{1y} = [w_1 / (1 - (C_f/C_e))]$$

$$w_1 = 0.6 - 0.4k = > 0.4$$

$$k = (M_{fx\text{Small}}) / (M_{fx\text{Large}}) = 0/100 = 0$$

$$w_1 = 0.6$$

$$C_{ey} = \pi^2 E I_y / L_y^2 = (\pi^2 \times 200000 \times 29.5 \times 10^6 / 2000^2) = 14558$$

$$U_{1x} = 0.6 / (1 - 500/14558) = 0.621$$

$$(500/3553) + (0.85 \times 0.60 \times 150/913.5) + (0.6 \times 0.621 \times 100/127.3) = 0.52$$

c) Lateral torsional buckling

$$C_r = \theta A_g F_y (1 + \lambda^{2n})^{(-1/n)} = 3553 \text{ kN}$$

$$M_u = w_2 \pi / L_x (E I_y G J + (E \pi / L_x)^2 I_y C_w)$$

$$w_2 = 1.75 + 1.05k + 0.3k = 1.75 < 2.5$$

$$M_u = (1.75 \times \pi / 2000) (200000 \times 29.5 \times 10^6 \times 7.7 \times 10^4 \times 7.81 \times 10^5 + (\pi \times 200000 / 2000) 229.5 \times 10^6 \times 2.55 \times 10^{12})^{1/2} = 7667 \text{ kN.m}$$

$$M_p = Z_x F_y = 1015 \text{ kN.m}$$

$$\text{Since } M_u > 0.67 M_p \rightarrow M_{rx} = 1.15 \theta M_p [1 - (0.28 M_p / M_u)] = 1012 \text{ kN.m}$$

$$M_{rx} \leq \theta M_p$$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_{1x} = [w_1 / (1 - (C_f/C_e))] = > 1.0$$

$$\text{since } 0.6 < 1.0 \rightarrow U_{1x} = 1.0$$

$$U_{1y} = [w_1 / (1 - (C_f/C_e))] = 0.621 \text{ (as calculated above)}$$

$$(500/3553) + (0.85 \times 1.0 \times 150/913.5) + (0.6 \times 0.621 \times 100/127.3) = 0.57$$

d) Bi-axial bending

$$M_{fx}/M_{rx} + M_{fy}/M_{ry} = (150/914) + (100/127) = 0.95 < 1.0$$

$$\% \text{ difference} = (1.04 - 0.95) / 1.04 * 100 = 8.6\%$$

As shown in the example, codes results are different by 8%. In the next section, same design procedures are shown for the length of 8000 mm and in the end the results are compared again.

5.1.2 Example2- I-Shaped Compact Section at Length of 8000 mm

LRFD:

Section Class:

$$\text{flange: } b_f/2t_f = (8.97 / (2 \times 0.585)) = 7.67$$

$$\text{flange compactness} = 0.56(E/F_y)^{1/2} = 0.56(29000/50)^{1/2} = 13.48$$

since $7.67 < 13.48 \rightarrow$ flange is compact

$$\text{web: } h/t_w = 52$$

$$\text{web compactness} = 1.49(E/F_y)^{1/2} = 1.49(29000/50)^{1/2} = 11.14$$

since $52 > 11.14 \rightarrow$ web is slender

Compression Strength:

$$P_c = \phi_c P_n = \phi_c A_g F_{cr}$$

Slenderness Ratio:

$$kL_x/r_x = (1.0 \times 39314.9 / 9.55) = 32.98$$

$$kL_y/r_y = (1.0 \times 314.96 / 1.87) = 168.43$$

max slenderness ratio = 168.43

$$\lambda_c = (kL_y/r_y \pi) (F_y/E)^{1/2} = (168.43/\pi) (50/29000)^{1/2} = 2.226$$

$$\text{since } \lambda_c > 1.5 \rightarrow F_{cr} = (0.877 / \lambda_c^2) F_y = (0.877 / 2.226^2) 50 = 8.8 \text{ ksi}$$

$$P_c = (0.85) (20.10 \text{ in}^2) (8.8 \text{ ksi}) = 151 \text{ kips}$$

$$P_r/P_c = 112.5/151 = 0.74 > 0.2 \rightarrow$$

$$P_r/P_c + (8/9) (M_{rx}/M_{cx}) + (M_{ry}/M_{cy}) \leq 1.0$$

Moment:

M_{cx} :

$$L_p = 300 r_y / F_y^{1/2} = (300 \times 1.87 / 50^{1/2}) = 78.6 \text{ in}$$

$$X_1 = (\pi / S_x) (E G J A^2)^{1/2} =$$

$$(\pi / 154) (29000 \times 11200 \times 1.9 \times 20.10 / 2)^{1/2} = 1606.6$$

$$X_2 = (4 C_w / I_y) (S_x / G J)^2 =$$

$$(4 \times 9430 / 70.40) (154 / (11200 \times 1.9))^2 = 0.028$$

$$L_r = (r_y X_1) / ((F_y - F_r) (1 + (1 + X_2 (F_y - F_r)^2)^{1/2})^{1/2}) = 206.3 \text{ in}$$

$L_b > L_r \rightarrow$

$$M_{nx} = \theta (\pi / L_b) (E I_y G J + (\pi E / L_b)^2 I_y C_w)^{1/2} \leq \theta M_p = 0.9 \times Z_x F_y$$

$$\text{Since } M_{nx} = 3127 \text{ kip.ft} > 664$$

$$M_{cx} = 664 \text{ kip.ft}$$

$$M_{cy} = 0.9 Z_y F_y / 12 = (0.9) (24.5) (50) / 12 = 93.7 \text{ kip.ft}$$

Interaction Formula:

$$P_r / 2P_c + (M_{rx} / M_{cx}) + (M_{ry} / M_{cy}) =$$

$$(112.5/151) + (8/9) (112.5/664) + (75/93.7) = 1.62 > 1.0$$

Design not Ok!

CSA:

Section Class:

$$\text{flange: } b_f / 2t_f = (228 / (2 \times 14.9)) = 7.67$$

$$\text{since } 7.67 < 145 / (F_y)^{1/2} = 7.8 \rightarrow \text{flange is Class 1}$$

$$\text{web: } h / t_w = (603 - 2 \times 14.9) / 10.5 = 54.6$$

$$\text{since } 52 < 1100 / (F_y)^{1/2} [1 - 0.39 C_f / (\theta A F_y)] = 58.8 \rightarrow \text{web is Class 1}$$

Section is Class 1

Interaction Formula:

$$C_f / C_r + 0.85 U_1 x M_{fx} / M_{rx} + \beta U_1 y M_{fy} / M_{ry} \leq 1.0$$

$$\beta = 0.6$$

a) Cross sectional strength:

$$C_r = \theta A_g F_y = (0.9) (13000) (350) = 4095 \text{ kN}$$

$$M_{rx} = \theta Z_x F_y = (0.9) (2.9 \times 10^6) (350) = 913.5 \text{ kN.m}$$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_1 x = U_1 y = 1.0$$

$$(500/4095) + (0.85 \times 1.0 \times 150/913.5) + (0.6 \times 1.0 \times 100/127.3) = 0.73 < 1.0$$

b) Overall member strength:

$$C_r = C_{r0} = \theta A_g F_y (1 + \lambda^{2n})^{(-1/n)}$$

$$kL_x/r_x = (1.0 \times 8000 / 243) = 32.9$$

$$kL_y/r_y = (1.0 \times 2000 / 47.7) = 167.7 \rightarrow \text{maximum slenderness ratio}$$

$$\lambda_0 = (kL_y/r_y) (F_y / \pi^2 E)^{1/2} = (167.7) (350 / 200000 \times \pi^2)^{1/2} = 2.233$$

$$C_{r0} = (0.9) (13000 \times 350) (1 + (2.233)^{2 \times 1.34})^{-1/1.34} = 756 \text{ kN}$$

$$M_{rx} = \theta Z_x F_y = (0.9) (2.9 \times 10^6) (350) = 913.5 \text{ kN.m}$$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_1 x = [w_1 / (1 - (C_f / C_e))]]$$

$$w_1 = 0.6 - 0.4k = 0.4$$

$$k = (M_{fx} \text{ Small}) / (M_{fx} \text{ Large}) = 0 / 150 = 0$$

$$w_1 = 0.6$$

$$C_{ex} = \pi^2 E I_x / L_x^2 = (\pi^2 \times 200000 \times 764 \times 10^6 / 8000^2) = 23564$$

$$U_{1x} = 0.6 / (1 - 500/23564) = 0.61$$

$$U_{1y} = [w_1 / (1 - (C_f/C_e))]]$$

$$w_1 = 0.6 - 0.4k = 0.4$$

$$k = (M_{fxSmall}) / (M_{fxLarge}) = 0/100 = 0$$

$$w_1 = 0.6$$

$$C_e = \pi^2 E I_{yx} / L_y^2 = (\pi^2 \times 200000 \times 29.5 \times 10^6 / 8000^2) = 910$$

$$U_{1x} = 0.6 / (1 - 500/910) = 1.33$$

$$(500/756) + (0.85 \times 0.61 \times 150/913.5) + (0.6 \times 1.33 \times 100/127.3) = 1.37$$

c) Lateral torsional buckling

$$C_r = \theta A_g F_y (1 + \lambda^{2n})^{(-1/n)} = 756 \text{ kN}$$

$$M_u = w_2 \pi / L_x (E I_y G J + (E \pi / L_x)^2 I_y C_w)$$

$$w_2 = 1.75 + 1.05k + 0.3k = 1.75 \leq 2.5$$

$$M_u = (1.75 \times \pi / 8000) (200000 \times 29.5 \times 10^6 \times 7.7 \times 10^4 \times 7.81 \times 10^5 + (\pi \times 200000 / 8000) 229.5 \times 10^6 \times 2.55 \times 10^{12})^{1/2} = 622 \text{ kN.m}$$

$$M_p = Z_x F_y = 1015 \text{ kN.m}$$

Since $M_u < 0.67 M_p \rightarrow M_{rx} = \theta M_u = 560 \text{ kN.m}$

$$M_{ry} = \theta Z_y F_y = (0.9) (4.04 \times 10^5) (350) = 127.3 \text{ kN.m}$$

$$U_{1x} = [w_1 / (1 - (C_f/C_e))]] > 1.0$$

since $0.6 < 1.0 \rightarrow U_{1x} = 1.0$

$$U_{1y} = [w_1 / (1 - (C_f/C_e))]] = 0.621 \text{ (as calculated above)}$$

$$(500/756) + (0.85 \times 1.0 \times 150/560) + (0.6 \times 1.33 \times 100/127.3) = 1.52$$

d) Bi-axial bending

$$M_{fx}/M_{rx} + M_{fy}/M_{ry} = (150/560) + (100/127) = 1.05 > 1.0$$

$\% \text{difference} = (1.62 - 1.52) / 1.62 \times 100 = 6.2\%$
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As noticed in the above examples, at length of 2000mm the codes' results were different by 8.6%. However, by changing the length from 2000 mm to 8000mm, the difference dropped by almost 2.4% and calculated to be 6.2%.

Moreover, as noticed in the equations, CSA checks the strength of a beam-column for cross-sectional strength, overall member strength, lateral torsional buckling and bi-axial bending whereas LRFD uses only two equations 4-17 and 418 based only on the ratio of the factored axial load to factored axial resistance. In other words, although the codes results were almost in agreement, CSA uses a more precise and detailed approach in determining beam-column strength than does LRFD.

6.0 Shear

The following equations are used to calculate the shear strength of stiffened and unstiffened webs in CSA and LRFD.

LRFD

$$\text{Shear Strength} = \theta_v V_n = 0.9 V_n \quad (6-1)$$

$$(h/t_w) \leq 187 (k_v / F_y)^{1/2} \quad V_n = 0.6 F_y A_w \quad (6-2)$$

$$187 (k_v / F_y)^{1/2} < (h/t_w) \leq 234 (k_v / F_y)^{1/2}$$

$$V_n = 0.6 F_y A_w 187 (k_v / F_y)^{1/2} / (h/t_w) \quad (6-3)$$

$$(h/t_w) > 234 (k_v / F_y)^{1/2} \quad V_n = A_w 26400 k_v / (h/t_w)^2 \quad (6-4)$$

$$\text{Stiffened Web:} \quad k_v = 5 + 5 / (a/h)^2 \quad (6-5)$$

$$\text{Unstiffened Web:} \quad k_v = 5.0$$

CSA

$$\text{Shear Strength} = V_r = \phi A_w F_s \quad (6-6)$$

$$(h/w) \leq 439(k_v / F_y)^{1/2} \quad F_s = 0.66 A_w F_y \quad (6-7)$$

$$439(k_v / F_y)^{1/2} < (h/w) \leq 502(k_v / F_y)^{1/2}$$

$$F_s = 290(k_v F_y)^{1/2} / (h/w) \quad (6-8)$$

$$502(k_v / F_y)^{1/2} < (h/w) \leq 621(k_v / F_y)$$

$$F_s = 290(k_v F_y)^{1/2} / (h/w) + (1 / (1 + (a/h)^2))^{1/2} (0.5 F_y - 0.866(180000 k_v / (h/w)^2)) \quad (6-9)$$

$$621(k_v / F_y)^{1/2} > (h/w)$$

$$F_s = 180000 / (h/w)^2 + (1 / (1 + (a/h)^2))^{1/2} (0.5 F_y - 0.866(180000 k_v / (h/w)^2)) \quad (6-10)$$

Stiffened Web:

$$(a/h) < 1.0 \quad k_v = 4 + 5.34 / (a/h)^2 \quad (6-11)$$

$$(a/h) \geq 1.0 \quad k_v = 5.34 + 4 / (a/h)^2 \quad (6-12)$$

Unstiffened Web: $k_v = 5.34$

Section 6.1, shows an example of an unstiffened member, in which the shear strength is calculated according to both codes and the results were compared.

6.1 Design of Members for Shear**6.1.1 Example: Unstiffened Member**

What is the Shear Strength of W36x135 (W920x201) in CSA and LRFD?

LRFD:

$w = 0.6$ in

$h = 34.76$ in

$A_w = (d - 2x_t) \times w = (33.97) (0.6) = 20.38$ in²

$h/w = 34.76 / 0.6 = 57.93$

$418 / (F_y)^{1/2} = 59.11$

since $h/w < 59.11 \rightarrow$ Unstiffened web, $k_v = 5.0$

$h/w < 187(k_v / F_y)^{1/2} = 59.13$

$V_n = 0.6 F_y A_w = 0.6 \times 50 \times 20.38 = 611.4$ kips

Shear strength = $0.9 V_n = 611.4 \times 0.9 = 550.3$ kips = 2448 kN

CSA:

$W = 15.2$ mm

$h = 882.9$ mm

$A_w = (862.8) (15.2) = 13115$ mm²

Unstiffened web $\rightarrow k_v = 5.34$

$$h/w = 882.9/15.2 = 58$$

$$439(k_v/F_y)^{1/2} = 54.2$$

$$502(k_v/F_y)^{1/2} = 62$$

since $h/w > 54.2$ but less than 62

$$F_s = F_s = 290(k_v/F_y)^{1/2}/(h/w) = 216.2 \text{ kN}$$

$$V_r = 0.9A_w F_s = 2552 \text{ kN}$$

$$\% \text{ difference} = (2552 - 2448)/2552 \times 100 = 4\%$$

as shown in the example, the difference between results is minimal and as a result it can be concluded that the shear design is the same in CSA and LRFD.

7.0 Conclusion

Limit States Design, adopted by both CSA and LRFD is a design method in which the performance of a structure is checked against collapse and unserviceability. As a result, to prevent the occurrence of such severe cases, multiple load factors and resistance factors are introduced in load and member resistance calculations to take into account the probability of any underestimation of loads and member resistance respectively. On the other hand, allowable stress design ensures that the stresses imposed on structural elements due to service loads do not exceed the elastic limit. As a result, only the resistance of structural elements is divided by a factor of safety.

Although, both codes are based on factored load and resistance factor principle, there still exist some minor differences in design of structural members. The differences were examined explicitly by means of various design examples for tension members, beam-

columns and compression members, and comparison graphs for flexural members.

According to comparison results, tension members and flexural members tend to give almost the same results and their only difference is due to the resistance factors used in each code. However, for compression members, the local buckling equations of slender structural elements tend to be different due to the fact that LRFD includes a non-dimensional reduction factor in local buckling equation whereas CSA doesn't. Finally, in design of beam-columns, there exist minor differences in results due to the fact that CSA uses a more detailed approach for checking the member strength by checking cross-sectional strength, overall member strength, lateral torsional buckling and bi-axial bending. However, LRFD uses only 2 different types of interaction formulas based on the ratio of the factored axial load to factored axial resistance of the beam-column. In the end, the minor difference in the result of shear design example of unstiffened members illustrates the fact that the codes are in agreement. Based on the above examples and comparisons, it can be concluded that CSA and LRFD approach for design of various structural members are in agreement.

References

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