Safety Concept

Summary

- first priority in the design process is to create a safe design
- codes are continuously changing to account for new techniques and findings
- Limit States Design Concept in Canada:

  Design is based on the probability and mode of failure, or limit of usefulness, and the probability of the occurrence and variation of the load.

- a structure is characterized by its overall resistance
- *mean value*

  $\bar{x} = \frac{1}{n}\Sigma x_i$ or $\bar{x} \Rightarrow m_x$ when $n \to \infty$

- *standard deviation*

  $s_x = \sqrt{\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)^2}$ or $s_x \Rightarrow \sigma_x$ when $n \to \infty$

- *coefficient of variation*

  $V = s_x / \bar{x}$ or $= \sigma_x / m_x$ when $n \to \infty$

- *distribution density function* $f_b(x)$
- *Gaussian (Normal) distribution*

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x - m_x}{\sigma_x}\right)^2\right]$$

- *failure probability*

  p_f (failure probability of 10^{-3} means, that out of 1000 realized projects, one is going to fail [safety index of 3.2])

- *global safety factor* $\gamma_o = \frac{m_r}{m_s}$, nominal safety factor $\gamma_p = \frac{r_p}{s_p}$
- *safety index* $\beta = \frac{m_z}{\sigma_z} = \frac{m_R - m_s}{\sqrt{\sigma_R^2 + \sigma_s^2}}$,

  where $m_z = m_R - m_s$ and $\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2}$

  a) incorporates of the past experience by calibrating this index with existing buildings
  b) takes into consideration the results of different types of action
  c) distinguishes between limit states pertaining to ultimate failure or serviceability conditions.

Limit State	Safety Index β		
	Degree of Damage		
	small	medium	severe
Serviceability	2.0	2.5	3.0
Ultimate	4.2	4.7	5.2

General

The safety requirements for the evaluation of a structure thus:

- should include all construction and design methods and all building materials
- should enable comparison of different designs
- should make use of all the existing experience and expertise
- should be practicable

The first priority in the design process is to create a safe design, usually be following recognized rules or codes. The codes are continuously changing in order to utilize current techniques of design and analysis. The concept used in Canada at this time is the limit states approach, which bases the design of a structure on the probability and mode of failure, or limit of usefulness, and the probability of the occurrence and variation of the load. Using this concept, the structure is characterized by its overall resistance, which leads to a better understanding of the structural behavior than working with arbitrary equivalent values which is the case with allowable stress design methods. For example, the failure of a column is described by the buckling resistance, rather than by an equivalent stress which relates to the buckling load.

Considering Theoretical Background[1]

an adequate number of samples, n, we know that the arithmetic mean value

$$\bar{x} = \frac{1}{n}\Sigma x_i \qquad \text{(S.1)}$$

whereas $\bar{x}$ becomes the mean m_x when $n \to \infty$

The standard deviation

$$s_x = \sqrt{\frac{1}{n}\Sigma\left(x_i - \bar{x}\right)^2} \qquad \text{(S.2)}$$

[1] Note: Only the simplest and necessary concepts will be explained here in order to understand the following derivations.

becomes σ_x when $n \to \infty$

The coefficient of variation for a population of $n \to \infty$ is

$$V = \frac{\sigma_x}{m_x} \qquad \text{(S.3)}$$

The figures below depict for the population ($n \to \infty$) a distribution density $f_b(x)$ of the characteristic value x. The mean value m_x represents the centroid of the area below the function $f_x(x)$. The standard deviation can be recognized as the radius of gyration as measured from m_x. The area beneath the function $f_x(x)$ is defined as **unity**.

For the distribution function $F_x(x)$ the following relationships are valid:

$$F_x(x) = \int_{-\infty}^{x} f_x(x)dx$$

or

$$\frac{dF_x(x)}{dx} = f_x(x) \qquad \text{(S.4)}$$

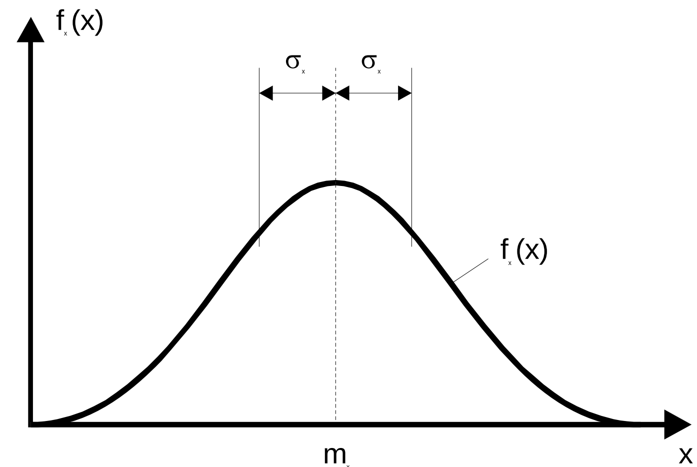

FIG. 7.1-1

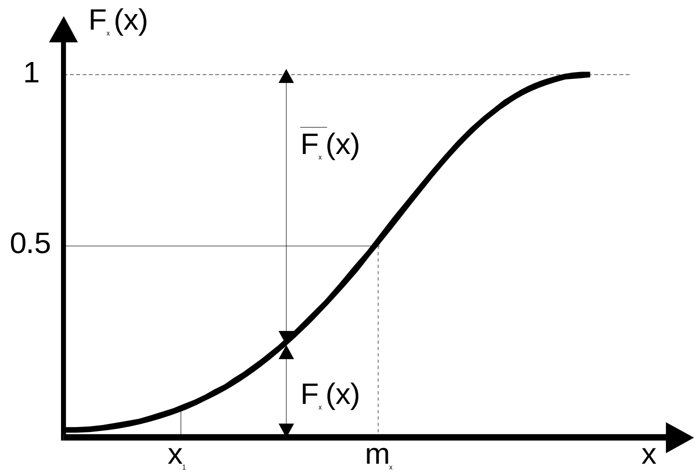

FIG. 7.1-2

The function value $F_x(x_1)$ indicates what fraction of the population is smaller or equal to the value x_1. It can be seen as the probability p_f that the value x is not larger than x_1. Thus, together with (S.4), we can derive the probability of an event to occur (i.e., the value of x to be smaller or equal to x_1):

$$p_f = F_x(x_1) = \int_{-\infty}^{x_1} f_x(x)dx \qquad \text{(S.5)}$$

Many different types of distribution are possible, e.g., normal distribution, logarithmic normal distribution, extreme distribution, etc. In the following derivations, we assume that the normal distribution adequately describes the characteristic of a population. For specialized investigations the following arguments can be adapted to other distribution types.

The Normal Distribution (Gaussian)

The equation for the normal distribution by Gauss depends on the values m_x and σ_x only:

$$f_x(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-m_x}{\sigma_x}\right)^2\right] \qquad \text{(S.6)}$$

This function cannot be integrated in closed form. The function values $f_x(x)$ and the integrals $F_x(x)$ are usually provided in tabular form. Often the concept of percentile x_p is used, e.g., the 5th percentile or the 95th percentile[2]. It has the meaning that p% of the events are expected to be below or above this value. In a normal distribution the relationship between the percentile value x_p (expressed as p%) and the standard deviation σ_x is as follows:

$$x_p = m_x \pm k.\sigma_x \qquad \text{(S.7)}$$

and can be tabulated as follows:

p(%)	k
20	0.842
10	1.282
5	1.645
2.50000	1.960
2.27500	2.000
1.00000	2.326
0.13500	3.000
0.00320	4.000
0.00003	5.000

[2] Note:
- The 5th percentile is thus: mean value minus 1.645 * (standard deviation).
- Mean value minus 2.0 * (standard deviation) corresponds to a 2.275th percentile.

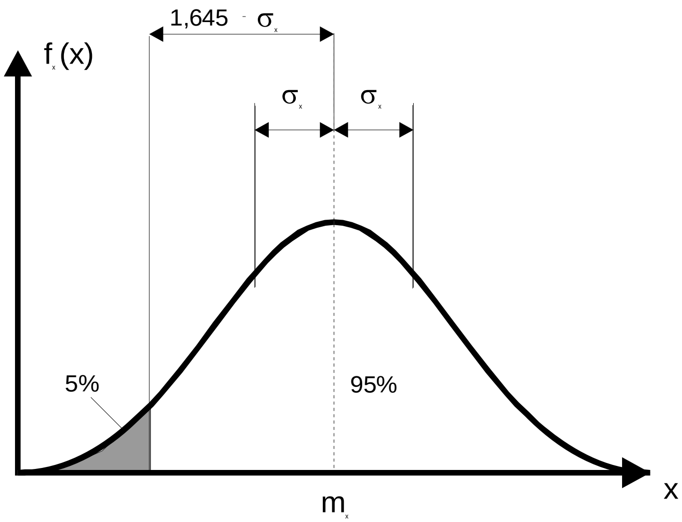

Fig. S.1-3: 5th-Percentile

The Probability of Failure

If we call S (stress) the distribution of an adequately large set of load readings and R (resistance) the corresponding set of failure loads of a type of structural component, then we can establish the following connection:

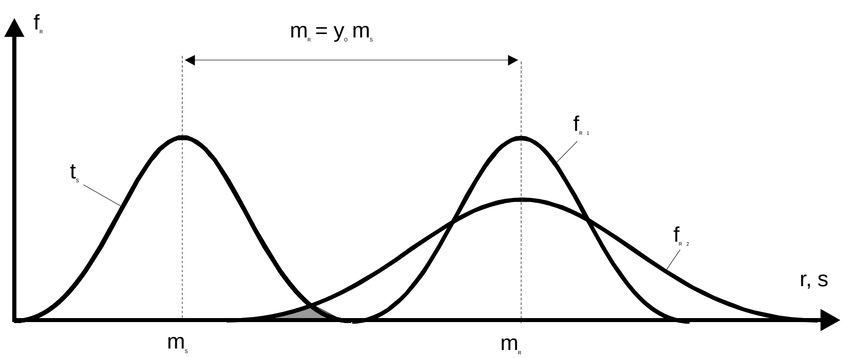

Fig. S.1-4: Global Safety Factor

Failure occurs when R < S. The hatched area in the figure is thus an indication of the probability of failure. We can now define the global safety factor

$$\gamma_o = \frac{m_f}{m_s}$$

From this simple example, we can draw the following conclusions:

- although the distance between the mean values m_R and m_s is the same, i.e., the same γ_o , the probability of failure greatly depends on the scatter of both S and R values, i.e., the magnitude of their respective standard deviations.
- thus, in order to achieve a consistent probability of failure, the global safety factor will have to be adjusted depending on the scatter (i.e., reliability) of load and resistant values. Exact knowledge of the distribution function in the asymptotic regions of the curves is essential, but unfortunately data for those regions is usually not available or is too sparsely spaced. With more complex distributions major numerical difficulties exist. There is just no mechanism to take into account the factor of experience.

Should one use the percentile values s_p, r_p instead of the mean values, the following relationship becomes apparent: For example, use the 95th percentile for the loads and the 5th percentile for the resistances.

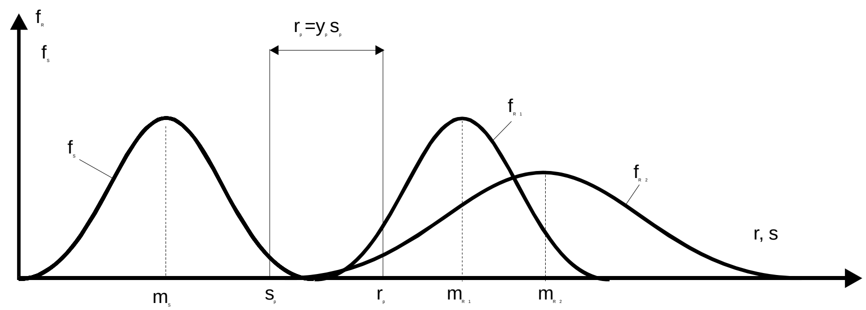

Fig. S.1-5: Nominal Safety Factor γ_p

As can be seen in the enlarged area, even if we define the nominal safety factor $\gamma_p = \frac{r_p}{s_p}$ to be dependent, not on the mean values, but on the 95th percentile of the loads and the 5th percentile of the resistances, the probability of failure is still dependent on the scatter of the distribution function. This probability is represented by the shaded areas. To quantity the concept of reliability or safety, the term probability of failure as defined above is thus not suitable. We will thus investigate the concept of a safety index.

Safety Index

Should one compare on deterministic basis the action S on a structure with the resistance R in a certain limit state (e.g., the ultimate limit state of failure), then the structure is deemed to be safe when R is chosen to be bigger than S. The difference

$$Z = R - S \qquad \text{(S.8)}$$

is defined as the safety margin. The defined limit state is reached when

$$R = S \quad \text{i.e., } Z = 0 \qquad \text{(S.9)}$$

Say, for instance, the action S and the resistance R are independent quantities with randomly scattered magnitudes with distribution densities $f_s(s), f_R(r)$, mean values of m_s, m_R and standard deviations σ_R. Then the safety margin Z is similarly distributed with a distribution density $f_z(z)$. Thus, when $f_s(s)$ and $f_R(r)$ are normally distributed, then $f_z(z)$ is also a normal distribution. Then the mean value

$$m_z = m_R - m_s \qquad \text{(S.10)}$$

and the standard deviation

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} \qquad \text{(S.11)}$$

The limit state equation (S.9) is thus valid for all the random values, s, r corresponding to the action and resistance for which

$$r = s \quad \text{, i.e.} \quad z = 0 \qquad \text{(S.12)}$$

As shown in the previous figure, cases of $Z \leq 0$ can hardly be avoided. The probability of occurrence for this violation, p_f, can be interpreted as the area under the distribution density curve $f_z(z)$ of the safety margin Z to the left of the vertical axis, i.e. $Z \leq 0$

$$\therefore p_f = \int_{-\infty}^{0} f_z(z)dz \qquad \text{(S.13)}$$

If we now define the quantity β such that

$$m_z = \beta\delta_z \qquad \text{(S.14)}$$

then the case of a failure can be interpreted as a percentile value of the distribution density function $f_z(z)$. In other words: Equal values for β mean equal degrees of reliability. An increasing β thus correlates with a higher reliability or higher safety factor. The value β is generally known as the **safety index**. From (S.10) and (S.11) we can derive

$$\beta = \frac{m_z}{\sigma_z} = \frac{m_R - m_s}{\sqrt{\sigma_R^2 + \sigma_s^2}} \qquad \text{(S.15)}$$

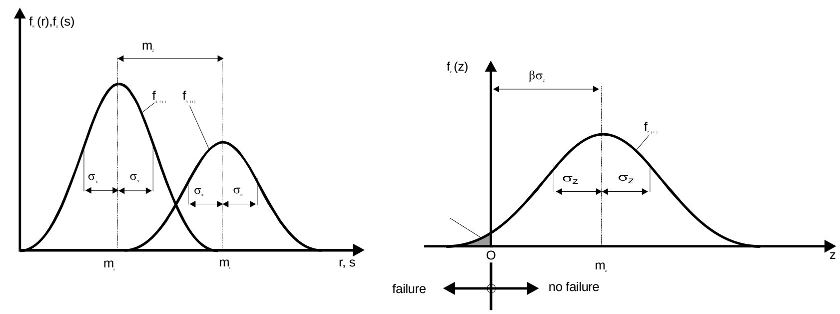

Fig. S.1-6: Distribution Density of Loads S, Resistance R and Safety Zone Z

From the derivation above it is apparent that the problem of the lack of knowledge about the fringe areas of the respective populations r and s has not be solved. The safety index, which flows from a simple arithmetic operation and which certainly only has a comparative meaning, can be used, however, for the purpose of:

- the incorporation of the past experience by calibrating this index with existing buildings
- to take into consideration the results of different types of action.
- to distinguish between limit states pertaining to ultimate failure or serviceability conditions.

By calculating β-values for existing structures that have been designed using common specifications (experience!), it can be assessed which β-values are being tolerated by the general public. The following numerical values seem to correspond with normal practice:

Limit State	Safety Index β		
	Degree of Damage		
	small	medium	severe
Serviceability	2.0	2.5	3.0
Ultimate	4.2	4.7	5.2

In linear problems (first order theory) and a normal distribution, the following relationship exists between the safety index β and the probability of failure p_f :

beta	p_f
5.2	10^{-7}
4.7	10^{-6}
4.2	10^{-5}
3.7	10^{-4}
3.2	10^{-3}

The failure probability represents the sum of all occurrences which have a negative value of the function $f_z(z)$. For every 1000 structures designed and built with a safety index of 3.2, one failure can be expected ($p_f = 10^{-3}$).

Relationship between Safety Index β and Safety Factor γ

The purpose of the design process is to maintain an acceptable difference between the action S and the resistance R (see figure below).
For obvious reasons, a high action percentile value is used.[3]

$$S_q = m_s + k_s\sigma_s \qquad \text{(S.16)}$$

e.g., mean plus 1.645 times standard deviation, i.e., 95th percentile.
For the resistance percentile, a low value is chosen, e.g., the 5th percentile with $k_R = 1.645$.

$$R_P = m_R - k_R\sigma_F \qquad \text{(S.17)}$$

[3] Only valid for normal distributions.

For other distribution curves, different relationships prevail.

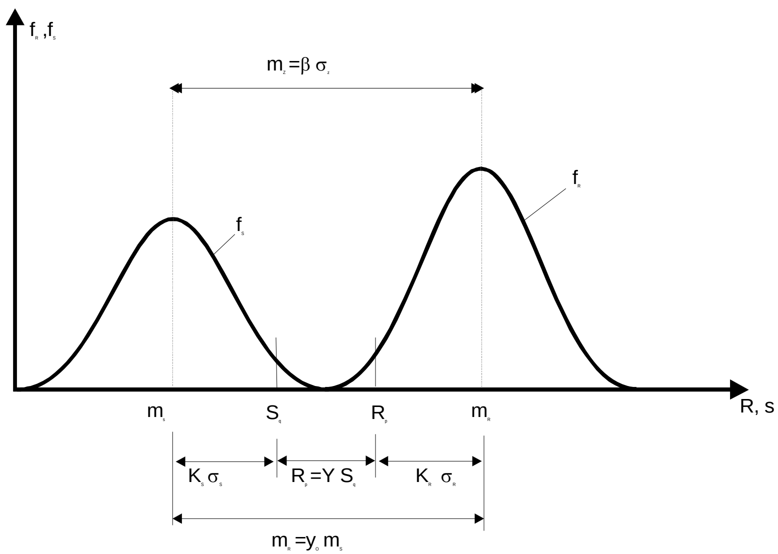

Fig. S.1-7: Safety Factors and Percentiles

For the global safety factor γ_o (derived from the means):

$$\gamma_o = \frac{m_R}{m_S} \qquad \text{(S.18)}$$

it follows, with (S.15) and (S.3)

$$\gamma_o = 1 + \beta\frac{\sqrt{\sigma_R^2 + \sigma_S^2}}{m_S}$$

$$= 1 + \beta\sqrt{\gamma_0^2\frac{\sigma_R^w}{m_R^w} + \frac{\sigma_S^2}{m_S^2}}$$

$$= 1 + \beta\sqrt{\gamma_o^2 V_S^2 + V_R^2}$$

which yields, after solving the quadratic equation in γ_o.

$$\gamma_o = \frac{1+\beta\sqrt{V_R^2+V_S^2-\beta V_R^2 V_S^2}}{1-\beta^2 V_R^2} \qquad (S.19)$$

This equation can be simplified significantly by applying the following "linearization" procedure:

$$\sqrt{\sigma_R^2+\sigma_S^2} = \alpha_R\sigma_R+\alpha_S\sigma_S \qquad (S.20)$$

Hence it follows from (S.15):

$$\beta = \frac{m_R-m_S}{\sqrt{\sigma_R^2+\sigma_S^2}} \qquad (S.21)$$

$$= \frac{m_R-m_S}{\alpha_R\sigma_R+\alpha_S\sigma_S}$$

and with equation (S.18):

$$\gamma_o = \frac{m_R}{m_S} \qquad (S.22)$$

$$= \frac{1+\beta\alpha_S V_S}{1-\beta\alpha_R V_R}$$

For the nominal safety factor γ, dependent on the percentile values S_q and R_p as per figure, and with (S.16) and 7.1.17):

$$\gamma_o = \frac{R_p}{S_q} \qquad (S.23)$$

$$= \frac{m_R-k_R\sigma_R}{m_S-k_S\sigma_S}$$

$$= \frac{m_R}{m_S}\frac{(1-k_R V_R)}{(1+k_S\sigma_S)}$$

Substituting (S.22):

$$\gamma = \frac{(1+\beta\alpha_S V_S)}{(1-\beta\alpha_R V_R)}\frac{(1-k_R V_R)}{(1-\beta\alpha_R V_R)} \qquad (S.24)$$

$$= V_S\gamma_R$$

yielding the partial safety factors

$$\gamma_S = \frac{1+\beta\alpha_S V_S}{1-k_S V_S} \qquad (S.25)$$

for the actions (e.g. loads)

$$\gamma_R = \frac{1-k_R V_R}{1+\beta\alpha_R V_R} \qquad (S.26)$$

for the resistance (e.g., material). By introducing these factors, the design process can be expressed as follows:

$$\gamma_S S_q \le \frac{R_p}{\gamma_R} \qquad (S.27)$$

or,

in other words, the overall nominal safety factor **γ** is split up into

- a partial safety factor $\boldsymbol{\gamma_S}$, which is multiplied with the characteristic value of the action (or percentile), which is independent of the world
- a partial safety factor $\boldsymbol{\gamma_R}$, which is divided into the characteristic value of the resistance (or percentile), and which, in turn, is independent of the action.

The probability of failure for different loading conditions is made more consistent by the use of distinct load factors for the different loads to which the structure is subject than in working stress design, where a single factor of safety is used. Furthermore, different resistance factors can, in parallel manner, be applied to determine member resistances with a uniform reliability. The combination of the load factor and the inverse of the resistance factor give a number comparable to a global factor of safety as used in former times. In the standard CAN3-S16-01 a resistance factor of 0.9 is generally used.

For live loads the load factor of 1.5 multiplied by the inverse of the resistance factor 1/0.9 which equals 1.67, is comparable to the working stress design standard. By using a load factor of 1.25 for dead load probabilistic studies indicate that consistent probabilities of failure are determined over all ranges of dead to live load ratios.

Note: In reality there is a dependence of the two partial safety factors, since the two values α_R and α_s are linked together ($\alpha^2_R + \alpha^2_S = 1$).

Errors in Design

References:

Blockley, D.I. (1980) The Nature of Structural Design and Safety, Ellis Horwood, Chichester.
Matousek, M. and Schneider,J. (1976) Untersuchungen zur Struktur des Sicherheitsproblems bei Bauwerken, Bereicht No. 59, Institut fir Baustatik und Konstruktion. Eidgenoessiche Techniche Hochschule, Zurich
Pugsley, A.G., (1973) The prediction of proneness to structural accidents, Structural Engineer, 51, (6) 195 – 196

Error factors in observed failure cases (Matousek 1976)

Factor	**%**
Ignorance, carelessness, negligence	35
Forgetfulness, errors, mistakes	9
Reliance upon others without sufficient control	6
Underestimation of influences	13
Insufficient knowledge	25
Objectively unknown situation	4
Remaining events	8

Prime Causes of Failures

Cause	**%**
Inadequate appreciation of loading conditions or structural behaviour	43
Mistakes in drawings or calculations	7
Inadequate information in contract documents or instructions	4
Contravention of requirements in contract documents or instructions	9
Inadequate execution of erection procedure	13
Unforeseen misuse, abuse and/or sabotage, catastrophe, deterioration	7
Random variation in loading, structure, materials, workmanship	10
Other	7

Pugsley (1973) suggested the following factors that affect the “proneness to structural accidents” as :

- new or unusual materials
- new or unusual methods of construction
- new or unusual types of structure
- experience and organization of design and construction teams
- research and development background
- financial climate
- industrial climate
- political climate.

Attempts have been made to provide a mathematical background to cover the errors and omissions that lead to failures. This is very difficult as the problem lies more in the area of social science than mathematics or engineering.

General Design Equation for Ultimate and Serviceability Limit State

Factored Resistance ≥ Factored Load

$$\phi R \geq \sum \alpha_i S_i$$

where,

ϕ	performance factor
R	the nominal member strength, or resistance
α	load factor
S	load effect (dead, live, etc.)(for values and details see applicable CAN/CSA Clause***: 7. Loads and Safety Criterion***)

The probability of failure for different loading conditions is made more consistent by the use of distinct load factors for the different loads than in working stress design where a single factor of safety is used. Different resistance factors can, in a similar manner, be applied to determine member resistances with a uniform overall reliability.

Serviceability refers to structural behaviour under actual loads, so-called nominal or specified loads. These loads have to be considered whenever there is some quantity that only depends on the actual weight of the material such as deflection, fatigue, vibration or camber of beams. Since the material properties of the material in the member are subject to the same reduction factor φ, it has to be included in Serviceability analysis or Limit States design.

Practical Concerns

For building design we are concerned with only a few loadings: **Dead, Live, Snow, Wind**. Occasionally there may be special loadings like machinery, or areas for parking trucks. For these loads one can look into bridge codes or obtain weights from other disciplines.

The total number of all these combinations of loadings can be quite large. The tables are shown below for reference. More detail can be found in Clause 6. of the HSC.

Table 11 Building Importance Factors for ULS and SLS (Cl. 6.2.2)

Importance category	Ultimate Limit States			Serviceability Limit States		
	Snow, I_s	Wind, I_w	Earthquake, I_E	Snow, I_s	Wind, I_w	Earthquake, I_E
Low	0.8	0.8	0.8	0.9	0.75	Check with the Natl. Bldg. Code and Commentary
Normal	1.0	1.0	1.0	0.9	0.75	
High	1.15	1.15	1.15	0.9	0.75	
Post-disaster	1.25	1.25	1.25	0.9	0.75	

Table 12 Building Importance Categories

Use and Occupancy	**Importance Category**
Low hazard to human life: barns, storage facilities, etc.	Low
All buildings other than those noted in this table	Normal
Post-disaster shelters or storage facilities for toxic materials	High
Post disaster buildings that have to be kept in operation: radio stations, etc.	Post-Disaster

Table 13 Load combinations for Ultimate Limit States

Case	**Load Combination**	
	Principal Loads	Companion Loads
1	1.4 D	
2	(1.25 D or 0.9 D) + 1.5L	0.5 S or 0.4 W
3	(1.25 D or 0.9 D) + 1.5S	0.5 L or 0.4 W
4	(1.25 D or 0.9 D) + 1.4 W	0.5 L or 0.5 S
5	1.0 D +1.0 E	0.5 L or 0.25 S

Note that this table shows that 15 load combinations have to be considered!

Errors and Omissions:

Attempts have been made to provide a mathematical background to cover the errors and omissions that lead to failures. This is very difficult as the problem lies more in the area of social science than mathematics or engineering. This is the reason for consultants to carry Professional insurance against such faults. In one situation a tower was built and after several years it developed cracks. By the time that the remediation had been done and a law suit went through the courts about 10 years had passed and the costs of the repairs and lawyers' fees far exceeded the value of the Liability insurance. The designer lost everything he owned.

Gimme Five

Limit States Design is based on:

- the probability of failure and failure mode [_]
- the probability of the occurrence and variation of load [_]
- the probability of the allowable stress design [_]

The global safety factor is defined as the ratio of:

- mean value / standard deviation [_]
- mean value of resistance / mean value of loads [_]
- coefficient of variation / mean value of resistance [_]

The percentile value of a distribution defines the:

- centre of the distribution curve [_]
- the standard deviation of the distribution curve [_]
- a tail ends of the distribution curve [_]

The Gaussion (Normal) distribution describes the following parameter correctly:

- the distribution of the load effects [_]
- the distribution of the temperature control [_]
- the distribution of the structural resistances [_]

For a safety index of 3.7, how many realized structures will result in one failure:

- 100,000 [_]
- 50,000 [_]
- 10,000 [_]