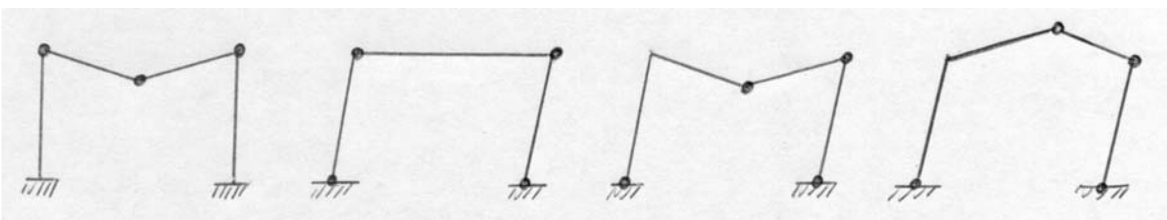
Mechanism or Virtual Work Method

The mechanism or the virtual work method is recommended to determine the collapse mechanism and the plastic moment diagram for more complex indeterminate frames and for those with more than one redundant.

The method is more general and is based on the upper bound theory of plastic design which states that the load corresponding to the assumed method of collapse will always be greater than, or at best equal to, the true ultimate load. The design procedure, therefore, is to assume a mechanism and compute the corresponding plastic moment. It may be necessary to assume several different mechanisms in order to be sure that the one with the largest required M_p has been tried.

In assuming trial mechanisms for single-storey frames, it is important to recognize that there are 4 distinct types:



Beam Mechanism   Sway Mechanism   Combined Mechanism   Gable Mechanism

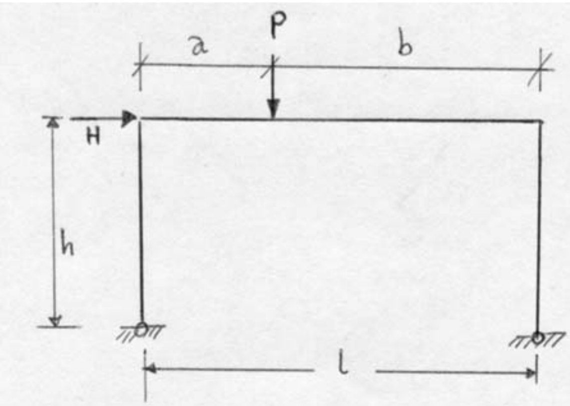
1. Beam Mechanism: it results when there is a high proportion of vertical to lateral load.
2. Sway Mechanism: this is sometimes referred to as a panel mechanism and is caused by large lateral forces applied to the top of the frame.
3. Combination Mechanism.
4. Gable Mechanism: this is a special case of the combination type and applies to gabled frames only.

The design procedure consists of determining the largest M_p for each of the three (or four) mechanism types. The largest of these then is the plastic moment for which the frame should be designed. The step-by-step procedure follows:

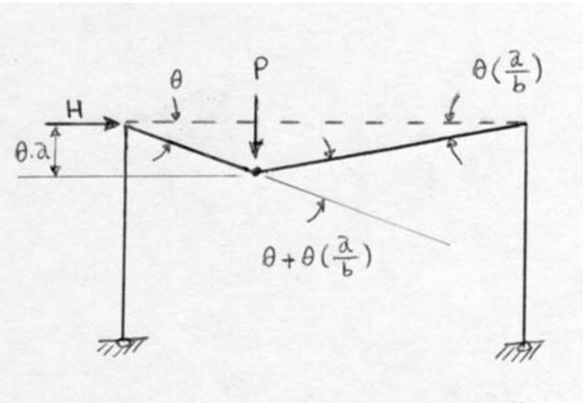
Step 1- Assume a specific collapse mechanism.
Step 2- Calculate the amount of internal virtual work which is defined as the sum of all the products of the plastic moments and their corresponding internal virtual angle changes. The internal virtual angle changes are computed by designating any one angle as θ and calculating all others in terms of θ and the geometry of the frame.
Step 3- Calculate the amount of external virtual work which is defined as the sum of all the external loads time the virtual distance through which they move at the collapse mechanism. This distance is calculated by recognizing that it is the product of the angle θ and the distance from the angle change to the load.

Step 4- Equate external to internal work. The angle θ cancels out and M_p can be solved for in terms of the loads and the dimensions of the frame.
Step 5- Sufficient collapse mechanisms are tried so that the designer is satisfied that one with the largest M_p has been found. This is done by drawing the plastic moment diagram for each trial mechanism to see that there are no moments larger than the plastic moment.

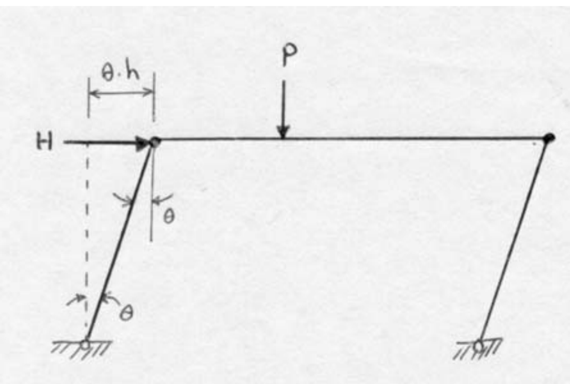
Example: **Portal frame with pin-ended columns, members of uniform cross section**



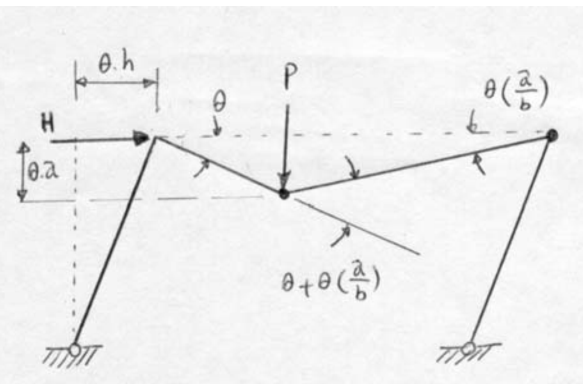
Frame: Dimension and Loadings



Mechanism 1: Beam Type



Mechanism 2: Sway Type



Mechanism 3: Combination Type

General solution without using a load factor (proportionality factor λ):

Mechanism 1 – Beam Type:

External work = $P \cdot \theta \cdot a$

Internal work = $M_p \cdot \theta + M_p \cdot \left(\theta + \dfrac{\theta \cdot a}{b}\right) + M_p \cdot \theta \cdot \left(\dfrac{a}{b}\right)$

External work = Internal work

$$P \cdot \theta \cdot a = M_p \cdot \theta \left[1 + 1 + \frac{a}{b} + \frac{a}{b}\right], \; a + b = l$$

$$M_{p1} = \frac{P.a.b}{2l}$$

Mechanism 2 – Sway Type:

External work = $H.\theta.h$

Internal work = $M_p.\theta + M_p.\theta$

$$M_{p2} = \frac{H.h}{2}$$

Mechanism 3 – Combination Type:

External work = $H.\theta.h + P.\theta.a$

Internal work = $M_p.\left[\theta + \theta.\frac{a}{b}\right] + M_p.\left(\theta.\frac{a}{b}\right)$

External work = Internal work

$$M_{p3} = \frac{b.(H.h + P.a)}{b + 2a}$$

The actual collapse mechanism will be the one with the largest required M_p. This example shows that it will depend on the value and location of the loads (H, P, a, b) and the dimensions and shape of the frame (l, h).