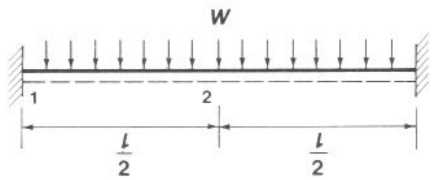
FIXED - ENDED BEAM WITH UNIFORMLY DISTRIBUTED LOAD

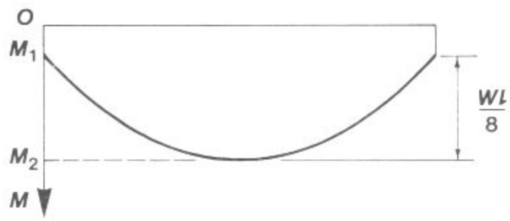
Note Title 3/29/2008

(a)



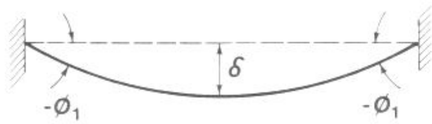
STATIC SYSTEM

(b)



BENDING MOMENT DISTRIBUTION (GENERAL FORMULATION)

(c)



DEFORMED SHAPE WITH END ROTATIONS PERMITTED (= HINGES ARE FORMING)

$$M_2 - M_1 = \frac{W\ell}{8}$$ (SEE GERE P.602 EQ. (9-14))

$$M_1 = -\frac{1}{12} W\ell - \frac{2EI\phi_1}{\ell}$$ (SEE GERE P.604 EQ. (9-19))

$$\delta = \frac{W\ell^3}{384\,EI} - \frac{1}{4}\ell\phi_1$$ (SEE GERE P.604 EQ. (9-18))

IF W IS SMALL ⇝ BEAM IS FULLY ELASTIC ⇝ $\phi_1 = 0$

⇝

$$M_1 = -\frac{1}{12} W\ell$$

$$M_2 = \frac{1}{24} W\ell$$

$$\delta = \frac{W\ell^3}{384\,EI}$$

(SEE GERE P.728 EQ. 10-32)

IF W IS LARGE ENOUGH THAT PLASTIC HINGES FORM,

$$\rightsquigarrow \quad M_1 = M_p$$

$$-\frac{1}{12} W_y \cdot \ell = M_p$$

$$W_y = \frac{12\,M_p}{\ell}$$

WITH W_y : "YIELD" LOAD

IF W INCREASES FROM W_y TO $W_y + \Delta W$, PLASTIC HINGES WILL UNDERGO ROTATION WHILE M_1 REMAINS CONSTANT:

$$W_y \rightsquigarrow W_y + \Delta W$$

$$M_1 \rightsquigarrow -M_p$$

$$\Delta M_2 = \frac{\Delta W\ell}{8}$$

$$\Delta M_1 = 0 = -\frac{1}{12}\Delta W\ell - \frac{2EI\Delta\phi_1}{\ell}$$

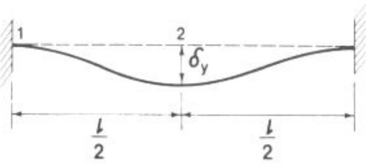
$$\Delta\delta = \frac{\Delta W\ell^3}{384\,EI} - \frac{1}{4}\ell\,\Delta\phi_1$$

(SEE ABOVE)

$$\Delta\phi_1 = -\frac{\Delta W \ell^2}{24EI}$$

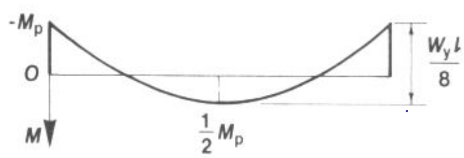
$$\Delta\delta = \frac{5\Delta W \ell^3}{384\,EI}$$

(a)



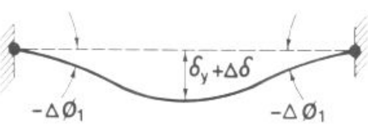
STATIC SYSTEM, $W = W_y$

(b)



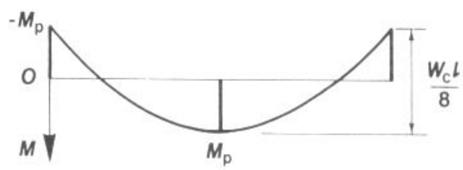
BENDING MOMENT DISTRIBUTION, $W = W_y$

(c)



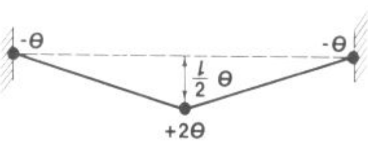
DEFLECTED SHAPE, $W = W_y + \Delta W$

(d)



INCREASE W_y BY ΔW TO W_c ⇝
BENDING MOMENT DISTRIBUTION, $W = W_c$

(e)



CHANGES OF GEOMETRY DURING COLLAPSE
= COLLAPSE MECHANISM

NOTE : CONDITIONS $\Delta M_1 = 0$, $\Delta\phi_1 \neq$ CORRESPOND TO SIMPLY SUPPORTED END CONDITIONS !

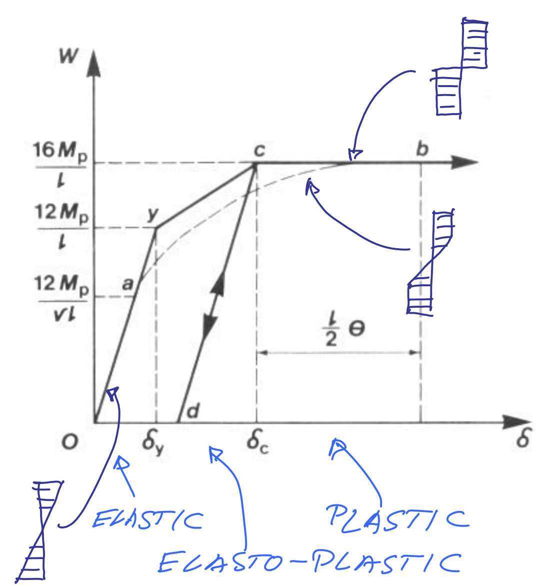
FROM BMD (a) → BMD (c) ⇝ INCREASE BY ΔW

$$0.5\,M_p + \frac{\Delta W \ell}{8} = M_p$$

$$\Delta W = \frac{4 M_p}{\ell}$$

$$\Delta\phi_1 = -\frac{M_p \ell}{6EI}$$

$$\Delta\delta = \frac{5 M_p \ell^2}{96\,EI}$$



$$\rightsquigarrow \quad W_c = 12\,M_p/\ell + 4\,M_p/\ell$$

$$= 16\,M_p/\ell$$

$$\nu = M_p/M_y = \text{"SHAPE FACTOR"}$$